Related-Key Almost Universal Hash Functions: Definitions, Constructions and Applications

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Outline

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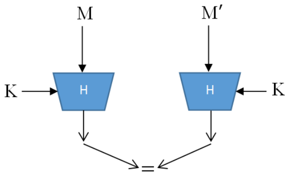
Universal hash functions

Almost Universal (AU)

$H : \mathcal{K} \times \mathcal{D} \rightarrow \mathcal{R}$



Definition (AU)

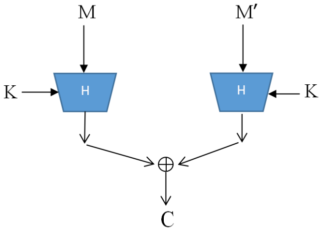
H is an ϵ-almost-universal (ϵ-AU) hash function, if for any $M, M' \in \mathcal{D}$, $M \neq M'$,

$$Pr[K \xleftarrow{\$} \mathcal{K} : H_K(M) = H_K(M')] \leq \epsilon$$

When ϵ is negligible, we say that H is AU.

Universal hash functions

Almost XOR Universal (AXU)



Definition (AXU)

Let $(\mathcal{R}, \oplus)$ *be an abelian group.* H *is an* ϵ*-almost-XOR- universal (*ϵ*-AXU), if for any* $M, M' \in \mathcal{D}$, $M \neq M'$ *and* $C \in \mathcal{R}$,

$$Pr[K \xleftarrow{\$} \mathcal{K} : H_K(M) \oplus H_K(M') = C] \leq \epsilon$$

When ϵ *is negligible, we say that* H *is AXU.*

Example

- $H_K(M) = MK$ $\quad$ $1/2^n$-AXU

$$H_K(M) \oplus H_K(M') = C$$

$$MK \oplus M'K = C$$

$$(M \oplus M')K = C$$

$$K = C(M \oplus M')^{-1}$$

$$\Pr[K \xleftarrow{\$} \mathcal{K} : H_K(M) \oplus H_K(M') = C] = 1/2^n$$

Example

- $H_K(M) = MK$
- $Poly : \{0,1\}^n \times \{0,1\}^{nm} \to \{0,1\}^n$,

$$Poly_K(M) = M_1K^m \oplus M_2K^{m-1} \oplus \cdots \oplus M_mK$$

$$M = M_1 \| M_2 \| \cdots \| M_m \in \{0,1\}^{nm}, M_i \in \{0,1\}^n, i = 1, \cdots, m.$$

$Poly$ is $m/2^n$-AXU.

Universal hash functions

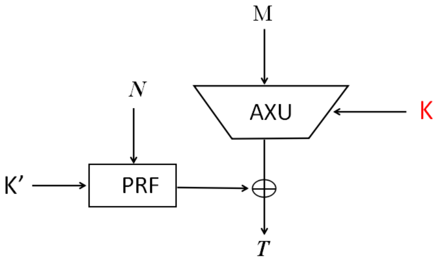
UHF-based schemes

- Message authentication code (MAC)
- Tweakable block cipher (TBC)
- Authenticated encryption (AE) scheme

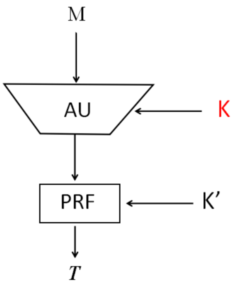
Universal hash functions

UHF-based schemes

- MAC



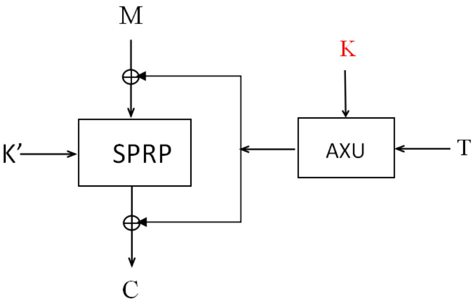
[Wegman and Carter, 1981]



[Brassard, 1982]

UHF-based schemes

- TBC



[Liskov et al., 2002]

Outline

Related-key attacks

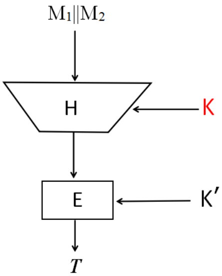
- firstly applied to block ciphers [Biham, 1993]
- Bellare and Kohno gave a formal definition of RKA-PRP and RKA-PRF [Bellare and Kohno, 2003]
- widely applied to MACs, TESes and AE schemes
  - Peyrin, et al: *Generic related-key attacks for HMAC*. ASIACRYPT 2012.
  - Dobraunig, et al: *Related-key forgeries for Prøst-OTR*. FSE 2015.
  - Sun, et al: *Weak-key and related-key analysis of hash-counter- hash tweakable enciphering schemes*. ACISP 2015.

Outline

RKA against UHF-based schemes

MAC

$Eg\ 1.\ H_K(M_1\|M_2) = M_1K^2 \oplus M_2K$ $\quad$ $2/2^n$-AXU



Query:

$$M\|M \xrightarrow{(K',\ K\oplus 1)} T$$

$$\begin{aligned} T &= E_{K'}(H_{K\oplus 1}(M\|M)) \\ &= E_{K'}(M(K\oplus 1)^2 \oplus M(K\oplus 1)) \\ &= E_{K'}(MK^2 \oplus MK) \end{aligned}$$

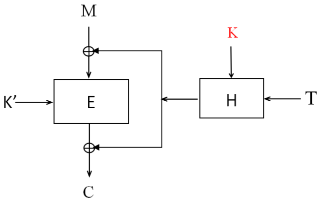
Forge:

$$M\|M \xrightarrow{(K',\ K)} T$$

RKA against UHF-based schemes

TBC

$Eg\ 2.\ \ H_K(T) = TK$ $\qquad$ $1/2^n$-AXU



Query:

$$(T, M) \xrightarrow{(K',\ K \oplus 1)} C$$

$$C = E_{K'}(M \oplus T(K \oplus 1)) \oplus T(K \oplus 1)$$

$$C \oplus T = E_{K'}((M \oplus T) \oplus TK) \oplus TK$$

Predict:

$$(T, M \oplus T) \xrightarrow{(K',\ K)} C \oplus T$$

Problems in the two-key schemes

$Eg\ 1.\quad H_{K\oplus 1}(M\|M) = H_K(M\|M)$

$Eg\ 2.\quad H_{K\oplus 1}(T) \oplus H_K(T) = T$

Collisions

The attacks have nothing to do with the block cipher.

RKA against UHF-based schemes

Almost all the existing two-key schemes which

based on universal hash function are

not related-key secure.

Outline

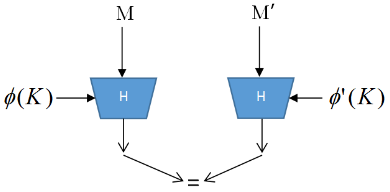
RKA-AU

Definition (RKA-AU)

H is an ϵ-related-key-almost-universal (ϵ-RKA-AU) hash function for the RKD set Φ, if $\forall \phi, \phi' \in \Phi$, $M, M' \in \mathcal{D}$, $(\phi, M) \neq (\phi', M')$,

$$Pr[K \xleftarrow{\$} \mathcal{K} : H_{\phi(K)}(M) = H_{\phi'(K)}(M')] \leq \epsilon.$$

When ϵ is negligible we say that H is RKA-AU.



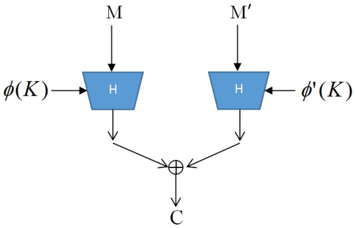
RKA-AXU

Definition (RKA-AXU)

Let $(\mathcal{R}, \oplus)$ be an abelian group. H is an ϵ-related-key-almost-XOR-universal (ϵ-RKA-AXU) hash function for the RKD set Φ, if $\forall \phi, \phi' \in \Phi$, $M, M' \in \mathcal{D}$, $(\phi, M) \neq (\phi', M')$ and $C \in \mathcal{R}$,

$$Pr[K \stackrel{\$}{\leftarrow} \mathcal{K} : H_{\phi(K)}(M) \oplus H_{\phi'(K)}(M') = C] \leq \epsilon.$$

When ϵ is negligible we say that H is RKA-AXU.



Restrictions on the RKD sets

Default RKD set

$$\Phi^{\oplus} = \{XOR_{\Delta} : K \mapsto K \oplus \Delta, \Delta \in \mathcal{K}\}$$

Not ideal functions

$Poly : \{0,1\}^n \times \{0,1\}^{nm} \to \{0,1\}^n,$

$$Poly_K(M) = M_1K^m \oplus M_2K^{m-1} \oplus \cdots M_mK$$

$M = M_1 \| M_2 \| \cdots \| M_m \in \{0,1\}^{nm}, M_i \in \{0,1\}^n, i = 1, 2, \cdots, m$

Let $M = M' = 0^{mn}$, $\phi \neq \phi'$.

$$Poly_{\phi(K)}(0^{mn}) = Poly_{\phi'(K)}(0^{mn}) = 0$$

$Poly$ is not RKA-AU.

Not ideal functions

Almost all the existing UHFs are not RKA-AU.

- MMH : $H_K(M) = (((\sum_{i=1}^{t} M_i K_i) \bmod 2^{64}) \bmod p) \bmod 2^{32}$, $M_i, K_i \in \mathbf{Z}_{2^{32}}$ and $p = 2^{32} + 15$;
- Square Hash : $H_K(M) = \sum_{i=1}^{t} (M_i + K_i)^2 \bmod p$, $M_i, K_i \in \mathbf{Z}_p$;
- NMH : $H_K(M) = (\sum_{i=1}^{t/2} (M_{2i-1} + K_{2i-1})(M_{2i} + K_{2i})) \bmod p$, $M_i, K_i \in \mathbf{Z}_{2^{32}}$, $p = 2^{32} + 15$;
- NH , WH ...

Outline

Constructions

- FIL-RKA-AXU: RH1
- VIL-RKA-AXU: RH2

FIL-RKA-AXU

$\mathrm{RH1} : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}^n,$

$$\mathrm{RH1}_K(M) = MK + K^3.$$

Proof. Let $F(K) = \mathrm{RH1}_{K\oplus\Delta_1}(M) \oplus \mathsf{RH1}_{K\oplus\Delta_2}(M')$.

$F(K) = (\Delta_1 \oplus \Delta_2)K^2 \oplus (\Delta_1^2\Delta_2^2 \oplus M \oplus M')K \oplus (\Delta_1^3 \oplus \Delta_2^3 \oplus M\Delta_1 \oplus M'\Delta_2).$

CASE 1. $\Delta_1 \neq \Delta_2$. $F(K) = C$ has 2 roots at most.
CASE 2. $\Delta_1 = \Delta_2$. Then $M \neq M'$. $F(K) = C$ has 1 root.

$$\Pr[K \xleftarrow{\$} \{0,1\}^n : F(K) = C] \leq 2/2^n.$$

RH1 is $2/2^n$-RKA-AXU over the RKD set $\Phi^{\oplus}$.

FIL-RKA-AXU

Remark from one of reviewers

$$\mathrm{RH1}_K(M) = MK + K^3.$$

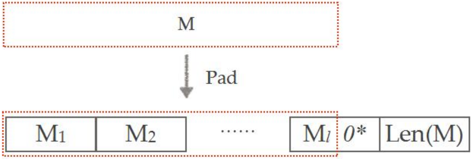
$\Phi_0 = \{id, f_\alpha\}$. $f_\alpha(K) = \alpha K,\ \alpha \in GF(2^n),\ \alpha^3 = 1.$

$$RH1_{f_\alpha(K)}(\alpha^{-1}M) = RH1_K(M)$$

RH1 is not RKA-AU over the RKD set Φ_0 .

VIL-RKA-AXU

$$Poly_K(M) = M_1K^m \oplus M_2K^{m-1} \oplus \cdots M_mK$$



$$pad(M) = M\|0^i\||M|$$

$Poly_K(pad(M))$ is VIL-AXU but not RKA-AXU.

VIL-RKA-AXU

$$Poly_K(M) = M_1K^m \oplus M_2K^{m-1} \oplus \cdots M_mK$$

$$\mathrm{RH2} : \{0,1\}^n \times \{0,1\}^* \to \{0,1\}^n,$$

$$\mathrm{RH2}_K(M) = \begin{cases} K^{l+2} \oplus Poly_K(pad(M)), & l\ is\ \ odd \\ K^{l+3} \oplus Poly_K(pad(M))K, & l\ is\ \ even \end{cases}$$

$$l = \lceil |M|/n \rceil + 1.$$

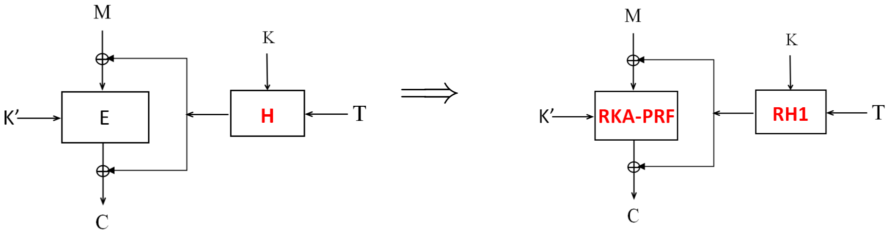
RH2 is $\mathrm{RKA\text{-}AXU}$ over the RKD set $\Phi^{\oplus}$.

RH2 VS Poly

$\mathrm{RH2}_K(M)$:	$Poly_K(pad(M))$
$T \leftarrow K^2$	$T \leftarrow 0$
for $i = 1$ **to** l	**for** $i = 1$ **to** l
$T \leftarrow (T \oplus M_i)K$	$T \leftarrow (T \oplus M_i)K$
if l is even	
$T \leftarrow TK$	
return T	**return** T

Solution

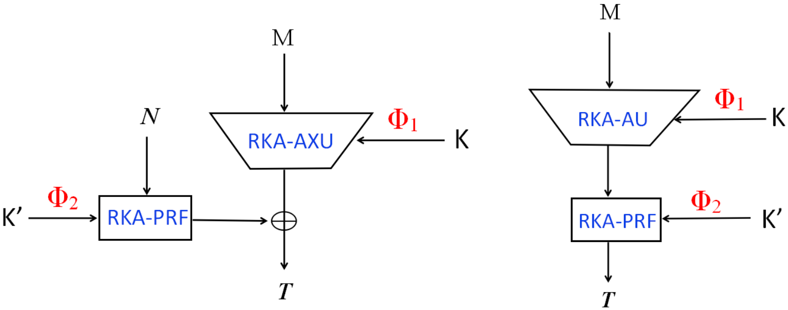
Whether it is secure ?



Outline

Applications

- related-key secure MACs



$\mathsf{MAC1}_{K,K'}(N, M) = H_K(M) \oplus F_{K'}(N)$

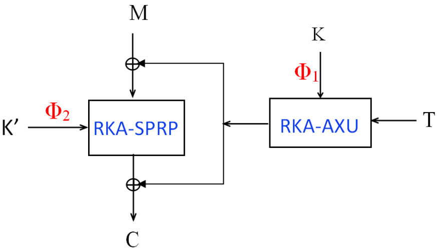
over $\Phi_1 \times \Phi_2$

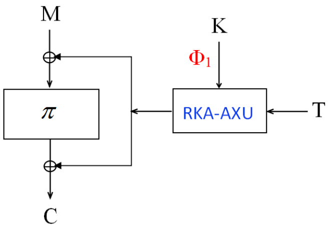
$\mathsf{MAC2}_{K,K'}(M) = F_{K'}(H_K(M))$

over $\Phi_1 \times \Phi_2$

Applications

- related-key secure MACs
- related-key secure TBCs





$\mathsf{TBC1}_{K,K'}(T,M) = E_{K'}(M \oplus H_K(T)) \oplus H_K(T)$

over $\Phi_1 \times \Phi_2$

$\mathsf{TBC2}_{K}(T,M) = \pi(M \oplus H_K(T)) \oplus H_K(T)$

over Φ_1

Conclusion

1. Propose a new concept of related-key almost universal hash function: RKA-AXU and RKA-AU.
2. Provide several efficient constructions named RH1, RH2 and RH3.
3. Show related-key secure MACs and TBCs, composed of RKA-AXU (RKA-AU) hash functions and other primitives such as RKA-PRPs and RKA-PRFs.

Thanks!

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