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# Improved Linear Hull Attack on Round-Reduced SIMON with Dynamic Key-guessing Techniques

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March 22, 2016

# Outline

- 1 Background and Contributions
- 2 Time Reduction in Linear Cryptanalysis
- 3 Improved Linear Hull Attack on Simon

# SIMON and Its Cryptanalysis Results

## Simon

- Proposed by NSA in 2013 (10 versions)
- Feistel structure (And (&), Rotation( $\lll$ ), Xor( $\oplus$ ))
- Optimal performance in hardware applications

## Existing cryptanalysis on Simon

- Differential attacks [Alkhzaimi et al.13, Abed et al.14, Biryukov et al.14, Wang et al.14]
- Linear (Hull) attacks [Abed et al.13, Alizadeh et al.14, Abdelraheem et al.14, Shi et al.14, Sun et al.14]
- Impossible Differential attacks [Alizadeh et al.14, Boura 14, Chen et al.15, Wang et al.+14]
- Zero-correlation attacks [Wang et al.+14]
- Integral attacks [Wang et al.+14]

# Motivation and Contributions

## Cryptanalysis on Simon

- Search better distinguishers
  - all kinds of automatic search for differential and linear distinguishers
  - connection between differential and linear characteristic
- Improve the attack procedure
  - differential attack with dynamic key-guessing technique by Wang *et al.*
  - **linear attack with dynamic key-guessing technique?**

## Contributions

- Propose the dynamic key-guessing technique for linear attack
  - Guess, Split and Combine
- Improve the linear hull attack on all versions of Simon
  - 23-round Simon32/64, . . . , 53-round Simon128/256
- Implement the 21-round attack on Simon32/64

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## General Framework of Matsui's Algorithm 2

$$P \underbrace{\xleftarrow{r_1}}_{K_P} X \boxed{\alpha \xleftrightarrow{r} \beta} Y \underbrace{\xrightarrow{r_2}}_{K_C} C$$

- Suppose

$$y = \alpha \cdot X \oplus \beta \cdot Y = F(P, C, K_P, K_C)$$

Then

$$c(K_P, K_C) = \sum_{P, C} (-1)^{F(P, C, K_P, K_C)} \quad (T = N * |K_P| * |K_C|)$$

- Further, let  $x = \text{compress}(P, C)$  is  $l_1$ -bit,  $k = \text{compress}(K_P, K_C)$  is  $l_2$ -bit and  $y = f(x, k)$ , the first step of linear cryptanalysis is to compute

$$B^k(y) = \sum_x (-1)^{f(x, k)} V[x] \quad (T = 2^{l_1 + l_2})$$

# General Framework of Matsui's Algorithm 2 - Linear Compression

- If  $y = f(x, k)$  is linear with some bits of  $x$  and  $k$ , for example

$$y = x_0 \oplus k_0 \oplus f_1(x', k'), x = x' || x_0, k = k' || k_0,$$

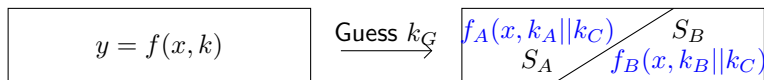
then  $x$  can be compressed further  $V'[x'] = V[x' || 0] - V[x' || 1]$ ,

$$B^k(y) = (-1)^{k_0} \sum_x (-1)^{f_1(x', k')} V'[x'] \quad (T = 2^{l_1+l_2-2})$$

- $k_0$  can be omitted (defined as the *related bit*)
- Multiple linear bits ( $\sum_{0 \leq i \leq t} k_i$  is the related bit)

$$\begin{aligned} y &= (x_0 \oplus k_0) \oplus \cdots \oplus (x_t \oplus k_t) \oplus f_t(x'', k'') \\ &= \left( \sum_{0 \leq i \leq t} x_i \right) \oplus \left( \sum_{0 \leq i \leq t} k_i \right) \oplus f_t(x'', k'') \end{aligned}$$

# Dynamic Key-guessing in Linear Attacks

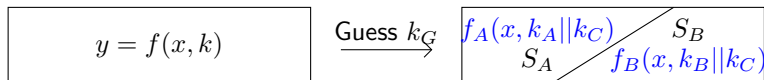


**Figure:** When  $k_G$  is known, the set of  $x$  can be splitted to two sets.  $f$  is independent of  $k_B$  in set  $S_A$  and independent of  $k_A$  in set  $S_B$ .

- For a boolean function  $y = f(x, k)$  and a counter vector  $V$ , computing  $B^k(y) = \sum_x (-1)^{f(x,k)} V[x]$  with a straight-forward method needs  $2^{l_1+l_2}$  calculations.
- If for different keys, the expression of  $y = f(x, k)$  is different, the key can be guessed dynamically.
  - Let  $k = k_G || k_A || k_B || k_C$  and the length is  $l_2^G, l_2^A, l_2^B, l_2^C$  respectively
  - When  $k_G$  is known, according to the value of  $x$  and  $k_G$ , the  $x$  can be splitted into two sets:  $S_A$  and  $S_B$
  - For  $x \in S_A$ ,  $y = f_A(x, k_A || k_C)$
  - For  $x \in S_B$ ,  $y = f_B(x, k_B || k_C)$



# Dynamic Key-guessing in Linear Attacks



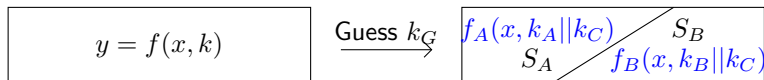
**Figure:** When  $k_G$  is known, the set of  $x$  can be splitted to two sets.  $f$  is independent of  $k_B$  in set  $S_A$  and independent of  $k_A$  in set  $S_B$ .

## The attack procedure and time complexity

- Guess  $k_G$ 
  - Split the  $x$  into two sets  $S_A, S_B$  according to the value of  $x$  and  $k_G$
  - Compute  $B^{k_A || k_C}(y) = \sum_{x \in S_A} (-1)^{f_A(x, k_A || k_C)}$  ( $T_A = N_A 2^{l_2^A + l_2^C}$ )
  - Compute  $B^{k_B || k_C}(y) = \sum_{x \in S_B} (-1)^{f_B(x, k_B || k_C)}$  ( $T_B = N_B 2^{l_2^B + l_2^C}$ )
  - Combine  $B^k(y) = B^{k_A || k_C}(y) + B^{k_B || k_C}(y)$  ( $T_C = 2^{l_2^A + l_2^B + l_2^C}$ )
- The total time is  $2^{l_2^G} (T_A + T_B + T_C)$ , which is

$$N_A 2^{l_2^G + l_2^A + l_2^C} + N_B 2^{l_2^G + l_2^B + l_2^C} + 2^{l_2} < 2^{l_1 + l_2}$$

# Dynamic Key-guessing in Linear Attacks



**Figure:** When  $k_G$  is known, the set of  $x$  can be splitted to two sets.  $f$  is independent of  $k_B$  in set  $S_A$  and independent of  $k_A$  in set  $S_B$ .

**Example:**  $y = f(x, k) = (x_0 \oplus k_0) \& (x_1 \oplus k_1)$ , compute the bias of  $y$  where  $V[x]$  denotes the number of  $x$

- GUESS  $k_0$  and SPLIT the  $x$  into two sets
  - For the  $x$  with  $x_0 = k_0$ , initialize a counter  $T_0$  and set  $T_0 = V[0|x_0] + V[1|x_0]$  (1 addition)
  - For the  $x$  with  $x_0 = k_0 \oplus 1$ , initialize a counter  $T_1$  and set  $T_1 = V[0|x_0] - V[1|x_0]$  (1 addition)
  - COMBINE  $B^k(y) = T_0 + (-1)^{k_1} T_1$  (2 additions)
- TIME:  $(2^2 \times 2^2 = 2^4) \rightarrow (2 \times (1 + 1 + 2) = 2^3)$

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## Brief Description of Simon

block size ( $2n$ )	key size ( $mn$ )	rounds
32 ( $n = 16$ )	64 ( $m = 4$ )	32
48 ( $n = 24$ )	72 ( $m = 3$ )	36
	96 ( $m = 4$ )	36
64 ( $n = 32$ )	96 ( $m = 3$ )	42
	128 ( $m = 4$ )	44
96 ( $n = 48$ )	96 ( $m = 2$ )	52
	144 ( $m = 3$ )	54
128 ( $n = 64$ )	128 ( $m = 2$ )	68
	192 ( $m = 3$ )	69
	256 ( $m = 4$ )	72

Table: The SIMON Family Block Ciphers

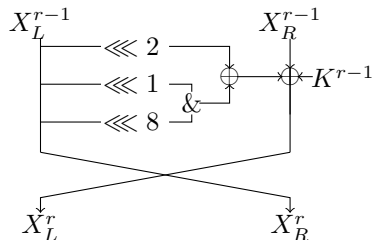


Figure: Round Function of SIMON

$$X_{L,i}^r = X_{i-2}^{r-1} \oplus (X_{i-1}^{r-1} \& X_{i-8}^{r-1}) \oplus X_{R,i}^{r-1} \oplus K_i^{r-1}$$

$$X_{R,i}^r = X_{L,i}^{r-1}$$

## Linear Hulls of Simon

BS	Input Active Bits	Output Active Bits	ALH	#R	Ref.
32	$X_{L,6}^i$	$X_{R,14}^{i+13}$	$2^{-31.69}$	13	[1]
	$X_{L,5}^i$	$X_{R,13}^{i+13}$	<b><math>2^{-30.19}</math></b>	<b>13</b>	<b>[2]</b>
	$X_{L,0}^i$	$X_{L,8}^{i+14}, X_{R,6}^{i+14}$	$2^{-32.56}$	14	[1]
48	$X_{L,7,11,19}^i, X_{R,9,17}^i$	$X_{L,5}^{i+15}, X_{R,3,7,11,19}^{i+15}$	$2^{-44.11}$	15	[1]
	$X_{L,6,14,18,22}^i, X_{R,16}^i$	$X_{L,4,20}^{i+15}, X_{R,6,18,20,22}^{i+15}$	$2^{-42.28}$	15	[2]
	<b><math>X_{L,1,5,21}^i, X_{R,23}^i</math></b>	<b><math>X_{L,1,5}^{i+16}, X_{R,23}^{i+16}</math></b>	<b><math>2^{-44.92}</math></b>	<b>16</b>	<b>[3]</b>
64	<b><math>X_{L,20,24}^i, X_{R,22}^i</math></b>	<b><math>X_{L,22}^{i+21}, X_{R,20,24}^{i+21}</math></b>	<b><math>2^{-62.53}</math></b>	<b>21</b>	<b>[1]</b>
	$X_{L,6}^i$	$X_{L,0}^{i+21}, X_{R,2,6,30}^{i+21}$	$2^{-60.72}$	21	[2]
	$X_{L,3,27,31}^i, X_{R,29}^i$	$X_{L,3}^{i+22}, X_{R,1,2}^{i+22}$	$2^{-63.83}$	22	[2]
96	<b><math>X_{L,2,34,38,42}^i, X_{R,36}^i</math></b>	<b><math>X_{L,2,42,46}^{i+30}, X_{R,0,40}^{i+30}</math></b>	<b><math>2^{-94.2}</math></b>	<b>30</b>	<b>[1]</b>
128	<b><math>X_{L,2,58,62}^i, X_{R,60}^i</math></b>	<b><math>X_{L,60}^{i+41}, X_{R,0,2,58,62}^{i+41}</math></b>	<b><math>2^{-126.6}</math></b>	<b>41</b>	<b>[1]</b>

\* BS means the block size of SIMON; #R means the number of rounds for the linear hull

[1] Mohamed Ahmed Abdelraheem *et al.* IACR Cryptology ePrint Archive 2014/681, 2014

[2] Danping Shi *et al.* IACR Cryptology ePrint Archive 2014/973, 2014

[3] Siwei Sun *et al.* IACR Cryptology ePrint Archive 2014/747, 2014

## Experimental Bias for the 13-round Linear Hull of Simon32

## Property 1 (Rotational Property)

If the potential of the linear hull  $(\alpha_L, \alpha_R) \xrightarrow{r\text{-round}} (\beta_L, \beta_R)$  for Simon $2n$  is  $\bar{\epsilon}^2$ , then for any  $i, 0 \leq i \leq n$ , the potential of

$$(\alpha_L \lll i, \alpha_R \lll i) \xrightarrow{r\text{-round}} (\beta_L \lll i, \beta_R \lll i)$$

for Simon $2n$  is also  $\bar{\epsilon}^2$ .

For Simon32:

$$X_{L,5}^i \rightarrow X_{R,13}^{i+13} \quad \bar{\epsilon}^2 = 2^{-30.19}$$

$$X_{L,6}^i \rightarrow X_{R,14}^{i+13} \quad \bar{\epsilon}^2 = 2^{-31.69}$$

Choose 600 random keys and compute the empirical bias

$\epsilon^2 =  p - 1/2 ^2$	Number/600
$\epsilon^2 \geq 2^{-27.19}$	0.012
$2^{27.19} > \epsilon^2 \geq 2^{-28.19}$	0.035
$2^{28.19} > \epsilon^2 \geq 2^{-29.19}$	0.097
$2^{29.19} > \epsilon^2 \geq 2^{-30.19}$	0.12
$2^{30.19} > \epsilon^2 \geq 2^{-31.19}$	0.173
$\epsilon^2 < 2^{-31.19}$	0.563

# Boolean representation of $X_{L,5}^i$ and $X_{R,13}^{i+13}$

- One round backward for  $X_{L,5}^i$

$$X_{L,5}^i = \underbrace{(X_{L,4}^{i-1} \& X_{L,13}^{i-1}) \oplus X_{L,3}^{i-1} \oplus X_{R,5}^{i-1}}_{x_0} \oplus \underbrace{K_5^{i-1}}_{k_0} = x_0 \oplus k_0.$$

- Four rounds backward for  $X_{L,5}^i$ :  $X_{L,5}^i = f(x, k)$  where

$$\begin{aligned} f(x, k) = & x_0 \oplus k_0 \oplus ((x_1 \oplus k_1) \& (x_2 \oplus k_2)) \oplus ((x_3 \oplus k_3) \& (x_4 \oplus k_4)) \oplus \\ & [(x_5 \oplus k_5 \oplus ((x_6 \oplus k_6) \& (x_7 \oplus k_7))) \& (x_8 \oplus k_8 \oplus ((x_9 \oplus k_9) \& (x_7 \oplus k_7)))] \oplus \\ & \{(x_{10} \oplus k_{10} \oplus ((x_6 \oplus k_6) \& (x_7 \oplus k_7))) \oplus \\ & [(x_{11} \oplus k_{11} \oplus ((x_{12} \oplus k_{12}) \& (x_{13} \oplus k_{13}))) \& (x_{14} \oplus k_{14} \oplus ((x_3 \oplus k_3) \& (x_{13} \oplus k_{13})))]\} \& \\ & (x_{15} \oplus k_{15} \oplus ((x_7 \oplus k_7) \& (x_9 \oplus k_9))) \oplus \\ & [(x_{14} \oplus k_{14} \oplus ((x_{13} \oplus k_{13}) \& (x_3 \oplus k_3))) \& (x_{16} \oplus k_{16} \oplus ((x_3 \oplus k_3) \& (x_4 \oplus k_4)))] \}, \end{aligned}$$

$x_0, \dots, x_{16}$  can be computed from  $X^{i-4}$ ,  $k_0, \dots, k_{16}$  are the xor-sums of some bits of  $K^{i-4}, \dots, K^{i-1}$  (See the following table)

- Four rounds forward for  $X_{R,13}^{i+13}$ : similar to that above

Table: 4 rounds before  $X_{L,5}^i$  for SIMON32

$x_0$	$X_{L,13}^{i-4} \oplus (X_{L,14}^{i-4} \& X_{L,7}^{i-4}) \oplus X_{R,15}^{i-4} \oplus X_{L,1}^{i-4} \oplus X_{L,5}^{i-4}$	$k_0$	$K_{15}^{i-4} \oplus K_1^{i-3} \oplus K_5^{i-3} \oplus K_3^{i-2} \oplus K_5^{i-1}$
$x_1$	$X_{L,14}^{i-4} \oplus (X_{L,15}^{i-4} \& X_{L,8}^{i-4}) \oplus X_{R,0}^{i-4}$	$k_1$	$K_0^{i-4}$
$x_2$	$X_{L,7}^{i-4} \oplus (X_{L,8}^{i-4} \& X_{L,1}^{i-4}) \oplus X_{R,9}^{i-4}$	$k_2$	$K_9^{i-4}$
$x_3$	$X_{L,2}^{i-4} \oplus (X_{L,3}^{i-4} \& X_{L,12}^{i-4}) \oplus X_{R,4}^{i-4}$	$k_3$	$K_4^{i-4}$
$x_4$	$X_{L,11}^{i-4} \oplus (X_{L,12}^{i-4} \& X_{L,5}^{i-4}) \oplus X_{R,13}^{i-4}$	$k_4$	$K_{13}^{i-4}$
$x_5$	$X_{L,14}^{i-4} \oplus (X_{L,15}^{i-4} \& X_{L,8}^{i-4}) \oplus X_{R,0}^{i-4} \oplus X_{L,2}^{i-4}$	$k_5$	$K_0^{i-4} \oplus K_2^{i-3}$
$x_6$	$X_{L,15}^{i-4} \oplus (X_{L,0}^{i-4} \& X_{L,9}^{i-4}) \oplus X_{R,1}^{i-4}$	$k_6$	$K_1^{i-4}$
$x_7$	$X_{L,8}^{i-4} \oplus (X_{L,9}^{i-4} \& X_{L,2}^{i-4}) \oplus X_{R,10}^{i-4}$	$k_7$	$K_{10}^{i-4}$
$x_8$	$X_{L,7}^{i-4} \oplus (X_{L,8}^{i-4} \& X_{L,1}^{i-4}) \oplus X_{R,9}^{i-4} \oplus X_{L,11}^{i-4}$	$k_8$	$K_9^{i-4} \oplus K_{11}^{i-3}$
$x_9$	$X_{L,1}^{i-4} \oplus (X_{L,2}^{i-4} \& X_{L,11}^{i-4}) \oplus X_{R,3}^{i-4}$	$k_9$	$K_3^{i-4}$
$x_{10}$	$X_{L,14}^{i-4} \oplus (X_{L,15}^{i-4} \& X_{L,8}^{i-4}) \oplus X_{R,0}^{i-4} \oplus (X_{L,3}^{i-4} \& X_{L,12}^{i-4}) \oplus X_{R,4}^{i-4}$	$k_{10}$	$K_0^{i-4} \oplus K_2^{i-3} \oplus K_4^{i-4} \oplus K_4^{i-2}$
$x_{11}$	$X_{L,15}^{i-4} \oplus (X_{L,0}^{i-4} \& X_{L,9}^{i-4}) \oplus X_{R,1}^{i-4} \oplus X_{L,3}^{i-4}$	$k_{11}$	$K_1^{i-4} \oplus K_3^{i-3}$
$x_{12}$	$X_{L,0}^{i-4} \oplus (X_{L,1}^{i-4} \& X_{L,10}^{i-4}) \oplus X_{R,2}^{i-4}$	$k_{12}$	$K_2^{i-4}$
$x_{13}$	$X_{L,9}^{i-4} \oplus (X_{L,10}^{i-4} \& X_{L,3}^{i-4}) \oplus X_{R,11}^{i-4}$	$k_{13}$	$K_{11}^{i-4}$
$x_{14}$	$X_{L,8}^{i-4} \oplus (X_{L,9}^{i-4} \& X_{L,2}^{i-4}) \oplus X_{R,10}^{i-4} \oplus X_{L,12}^{i-4}$	$k_{14}$	$K_{10}^{i-4} \oplus K_{12}^{i-3}$
$x_{15}$	$X_{L,7}^{i-4} \oplus (X_{L,8}^{i-4} \& X_{L,1}^{i-4}) \oplus X_{R,9}^{i-4} \oplus (X_{L,12}^{i-4} \& X_{L,5}^{i-4}) \oplus X_{R,13}^{i-4}$	$k_{15}$	$K_9^{i-4} \oplus K_{11}^{i-3} \oplus K_{13}^{i-4} \oplus K_{13}^{i-2}$
$x_{16}$	$X_{L,1}^{i-4} \oplus (X_{L,2}^{i-4} \& X_{L,11}^{i-4}) \oplus X_{R,3}^{i-4} \oplus X_{L,5}^{i-4}$	$k_{16}$	$K_3^{i-4} \oplus K_5^{i-3}$

Notice:  $x_{10} = x_3 \oplus x_5$ ,  $x_{15} = x_4 \oplus x_8$



# Compute the correlation of $f$

- 15 independent bits for  $x$  since  $x_{10} = x_3 \oplus x_5, x_{15} = x_4 \oplus x_8$
- 17 key bits of  $k$
- Compress  $x_0$  since  $f(x, k)$  is linear with  $x_0 \oplus k_0$ , 14 independent bits of  $x$  remain; 16 bits of  $k$  remain
- Guess  $k_1, k_3, k_7$ . CASE  $(x_1 \oplus k_1, x_3 \oplus k_3, x_7 \oplus k_7)$

- $(0, 0, 0)$

- $f = f_{00}$

$$f_{00} = ((x_5 \oplus k_5) \& (x_8 \oplus k_8)) \oplus \{ (x_{10} \oplus k_{10} \oplus [(x_{11} \oplus k_{11} \oplus ((x_{12} \oplus k_{12}) \& (x_{13} \oplus k_{13}))) \& (x_{14} \oplus k_{14})]) \& (x_{15} \oplus k_{15} \oplus [(x_{14} \oplus k_{14}) \& (x_{16} \oplus k_{16})]) \}$$

- 8 independent bits of  $x$  remain ( $x_{10} = x_3 \oplus x_5$ )
- 7 additions to compress the other bits of  $x$
- Getting new counters needs  $T_s = 2^8 \times 7$  additions
- $(0, 0, 1), \dots, (1, 1, 1)$  similar to  $(0, 0, 0)$

# Compute the correlation of $f_{00}$

- Guess  $k_5, k_{14}$  and split the  $x$  into 4 sets

Table: Simplification for  $f_{00}$  after guessing  $k_5, k_{14}$

Guess	Value	$f_{00}$	RB
$k_5, k_{14}$	0,0	$(x_{10} \oplus k_{10}) \& (x_{15} \oplus k_{15})$	
	0,1	$(x_{10,11} \oplus k_{10,11} \oplus ((x_{12} \oplus k_{12}) \& (x_{13} \oplus k_{13}))) \& (x_{15,16} \oplus k_{15,16})$	
	1,0	$(x_{10} \oplus k_{10}) \& (x_{15} \oplus k_{15})$	$k_8$
	1,1	$(x_{10,11} \oplus k_{10,11} \oplus ((x_{12} \oplus k_{12}) \& (x_{13} \oplus k_{13}))) \& (x_{15,16} \oplus k_{15,16})$	$k_8$

- Case  $(0, 0), (1, 0)$ 
  - Getting new counters:  $2^6 - 2$  additions
  - Similar to the example, however there is dependence between  $x_5$  and  $x_{10}$ , computing the bias needs no more than  $2^2$  additions
- Case  $(0, 1), (1, 1)$ 
  - Getting new counters:  $2^6 - 2^4$  additions
  - Computing the bias needs about  $2^{5.64}$  additions
- Total time  $2^{11.19}$  additions

# Attacks on Simon32/64

- Time of computing correlation of  $f$  is about

$$\left( \underbrace{(2^8 \times 7)}_{T_{split}} + \underbrace{2^{11.19}}_{T_{inner}} \right) \times 8 + \underbrace{2^{13} \times 7}_{T_{combine}} \times 2^3 = 2^{19.46}$$

- Data  $N = 2^{\bar{\epsilon}^{-2}} = 2^{31.19}$ , advantage  $a = 8$
- 21-round (4+13+4) attack:  $2^{35.84} \text{ Additions} + 2^{56} \text{ Encryptions}$ 
  - computing the bias:  $\underbrace{2^{14} \times 2^{19.46}}_{\text{head part}} + \underbrace{2^{16} \times 2^{19.46}}_{\text{tail part}} = 2^{35.84} \text{ Additions}$
  - exhaustive part:  $2^{64-a} = 2^{56} \text{ Encryptions}$
- 22-round (4+13+5) attack:  $2^{48.84} \text{ Additions} + 2^{56} \text{ Encryptions}$
- 23-round (5+13+5) attack:  $2^{61.84} \text{ Additions} + 2^{56.3} \text{ Encryptions}$

## Implementation of the 21-round Attack on Simon32

Table: Expected success probability using  $2^{31.19}$  data and advantage  $a = 8$ 

$\epsilon^2 =  p - 1/2 ^2$	proportion	$Pr_{succ}$ [SB02]
$\epsilon^2 \geq 2^{-27.19}$	0.012	$p_0 \approx 1.000$
$2^{27.19} > \epsilon^2 \geq 2^{-28.19}$	0.035	$1 > p_1 > 0.997$
$2^{28.19} > \epsilon^2 \geq 2^{-29.19}$	0.097	$0.997 > p_2 > 0.846$
$2^{29.19} > \epsilon^2 \geq 2^{-30.19}$	0.12	$0.846 > p_3 > 0.477$
$2^{30.19} > \epsilon^2 \geq 2^{-31.19}$	0.173	$0.477 > p_3 > 0.188$
$\epsilon^2 < 2^{-31.19}$	0.563	negligible
Expected success probability		$0.33 > Pr > 0.22$

- Experimental success probability: 0.277 (1000 experiments done)
- Data collection: 11 minutes (encrypt and compress  $2^{31.19}$  data)
- Key recovery: 11 minutes (get  $2^{32-a} = 2^{24}$  key candidates)
- About 5GB memory
- source code (<http://eprint.iacr.org/2015/666.pdf>)

## Summary of Linear Hull Attacks on SIMON

Cipher	Attacked rounds	Data	Time	Reference
SIMON32/64	21	$2^{30.56}$	$2^{55.56}$	Abdelraheem et al
	21	-	-	Shi et al
	<b>23</b>	<b><math>2^{31.19}</math></b>	<b><math>2^{61.84} A + 2^{56.3} E</math></b>	<b>Our result</b>
SIMON48/72	20	$2^{44.11}$	$2^{70.61}$	Abdelraheem et al
	<b>24</b>	<b><math>2^{47.92}</math></b>	<b><math>2^{67.89} A + 2^{65.34} E</math></b>	<b>Our result</b>
SIMON48/96	21	$2^{44.11}$	$2^{70.61}$	Abdelraheem et al
	21	-	-	Shi et al
	23	$2^{47.92}$	$2^{92.92}$	Sun et al
	<b>25</b>	<b><math>2^{47.92}</math></b>	<b><math>2^{89.89} A + 2^{88.28} E</math></b>	<b>Our result</b>
SIMON64/96	27	$2^{62.53}$	$2^{88.53}$	Abdelraheem et al
	<b>30</b>	<b><math>2^{63.53}</math></b>	<b><math>2^{93.62} A + 2^{88.13} E</math></b>	<b>Our result</b>
SIMON64/128	29	$2^{62.53}$	$2^{123.53}$	Abdelraheem et al
	29	-	-	Shi et al
	<b>31</b>	<b><math>2^{63.53}</math></b>	<b><math>2^{119.62} A + 2^{120.00} E</math></b>	<b>Our result</b>
SIMON96/96	<b>37</b>	<b><math>2^{95.2}</math></b>	<b><math>2^{67.94} A + 2^{88} E</math></b>	<b>Our result</b>
SIMON96/144	36	$2^{94.2}$	$2^{123.5}$	Abdelraheem et al
	<b>38</b>	<b><math>2^{95.2}</math></b>	<b><math>2^{98.94} A + 2^{136.00} E</math></b>	<b>Our result</b>
SIMON128/128	<b>49</b>	<b><math>2^{127.6}</math></b>	<b><math>2^{87.77} A + 2^{120} E</math></b>	<b>Our result</b>
SIMON128/192	48	$2^{126.6}$	$2^{187.6}$	Abdelraheem et al
	<b>51</b>	<b><math>2^{127.6}</math></b>	<b><math>2^{155.77} A + 2^{184.00} E</math></b>	<b>Our result</b>
SIMON128/256	50	$2^{126.6}$	$2^{242.6}$	Abdelraheem et al
	<b>53</b>	<b><math>2^{127.6}</math></b>	<b><math>2^{239.77} A + 2^{248.01} E</math></b>	<b>Our result</b>

\* '-' means not given;  $A$  means addition;  $E$  means encryption;

**Thanks for Your Attention**