Modeling Random Oracles under Unpredictable Queries

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23rd Fast Software Encryption
Nordrhein-Westfalen

The Random-Oracle Model (ROM)

- Random oracles (ROs) model ideal hash functions [BR93].
  In the RO model:

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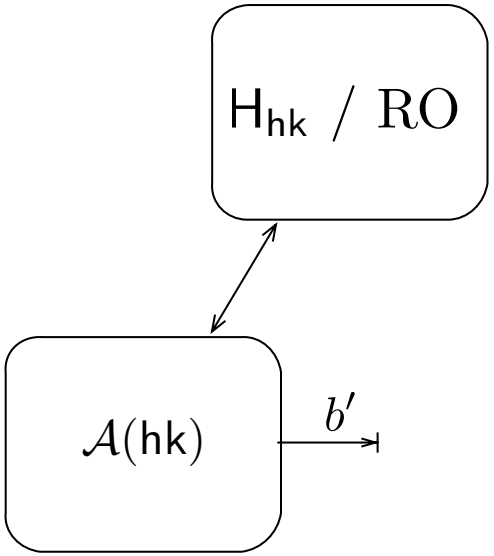
- ROs enable the security proofs of a wide range of practical and strongly secure cryptosystems: encryption & signature schemes, key exchange, disk encryption, ...
- Reliance on ROM, although practical, is also debatable:
  - There are **uninstantiable** ROM schemes [CGH98]:

    $\exists\, \mathsf{Enc}^{\mathcal{O}}$: $\mathsf{Enc}^{\mathcal{RO}}$ is secure but $\mathsf{Enc}^{\mathsf{H}}$ is insecure for **any** H.

  - Lack of a **definition** formalizing "RO-like behavior."

(Very) Naïve Attempt at Modeling ROs

Call a hash function “IND-RO” if, over a random choice of hk:



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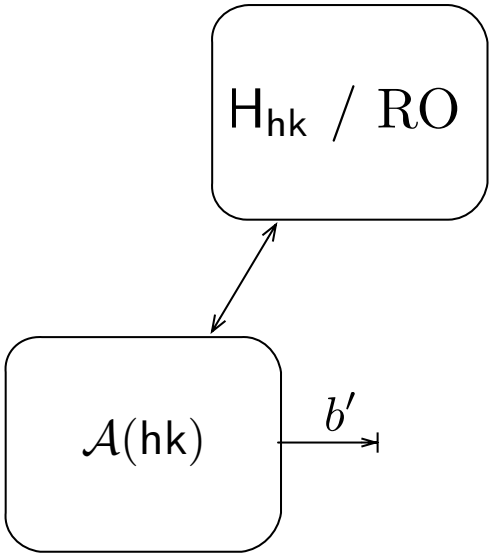

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Clearly uninstantiable:

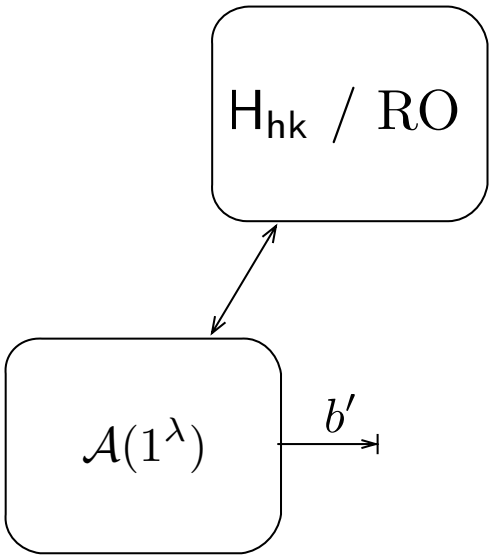
$\mathcal{A}(\mathsf{hk})$: compute $\mathsf{H}_{\mathsf{hk}}(0)$ and compare to the oracle's answer.

But observe:

The attack works because the "full" input $(\mathsf{hk}, 0)$ is known to $\mathcal{A}$.

Can We Fix the Naïve Model?

Let's hide hk.



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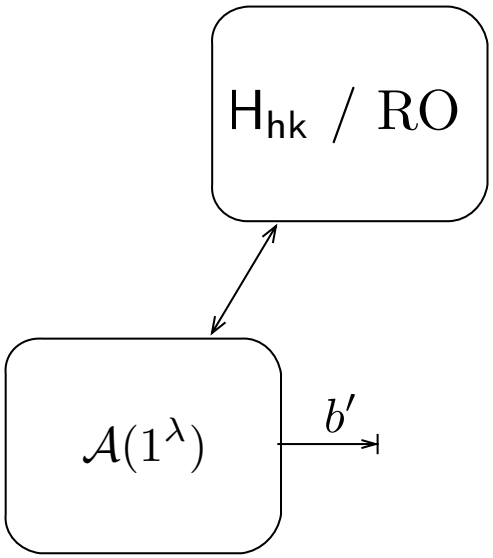

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First idea:

Split $\mathcal{A}$: one part gets hk and the other gets oracle access.

Modeling ROs via Split Adversaries

Call the two components of $\mathcal{A}$ the **source** $\mathcal{S}$ and the **distinguisher** $\mathcal{D}$:

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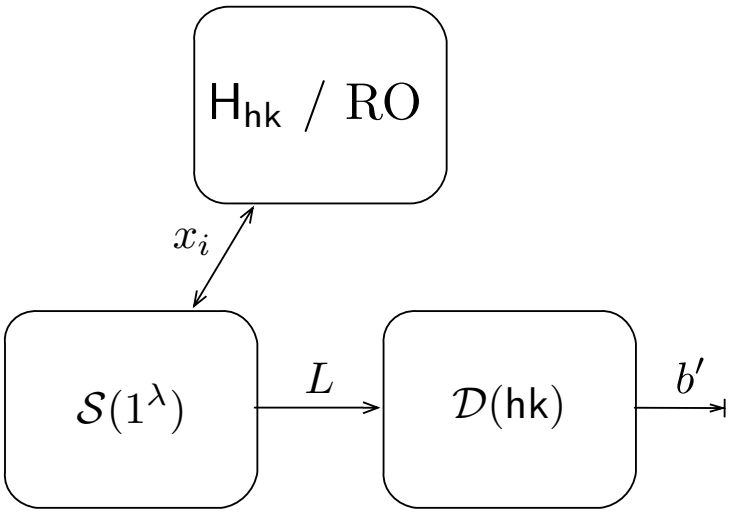

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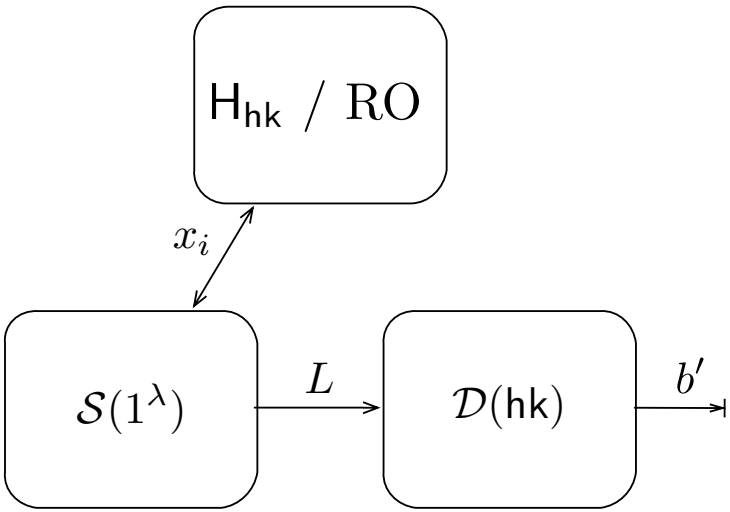

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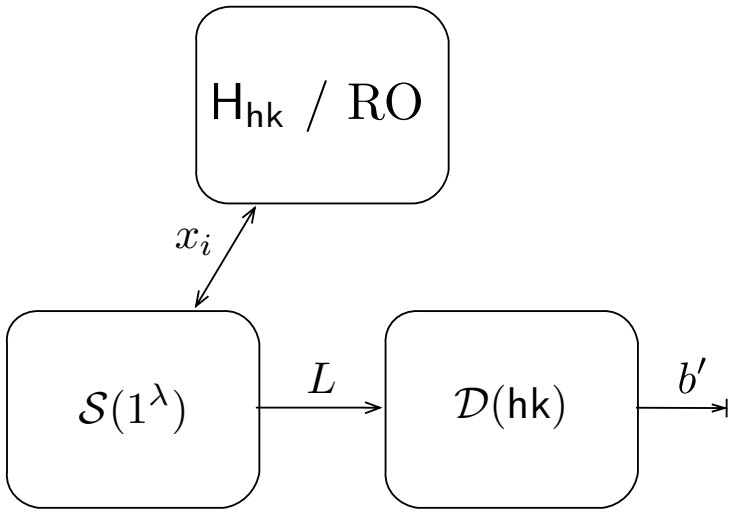

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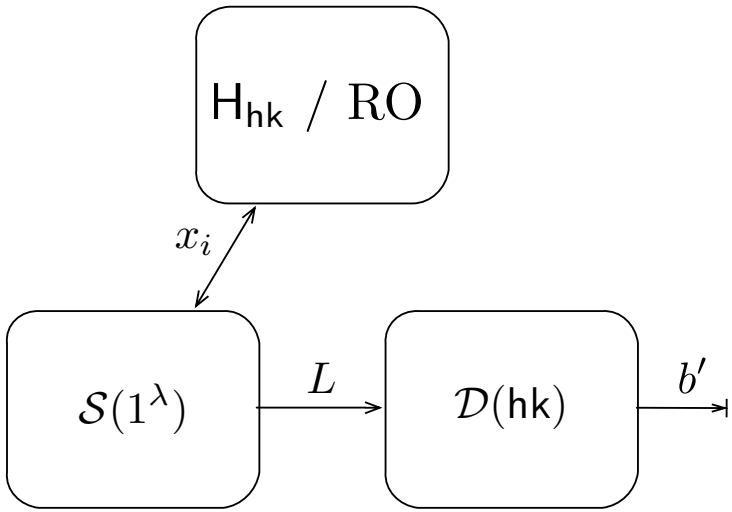

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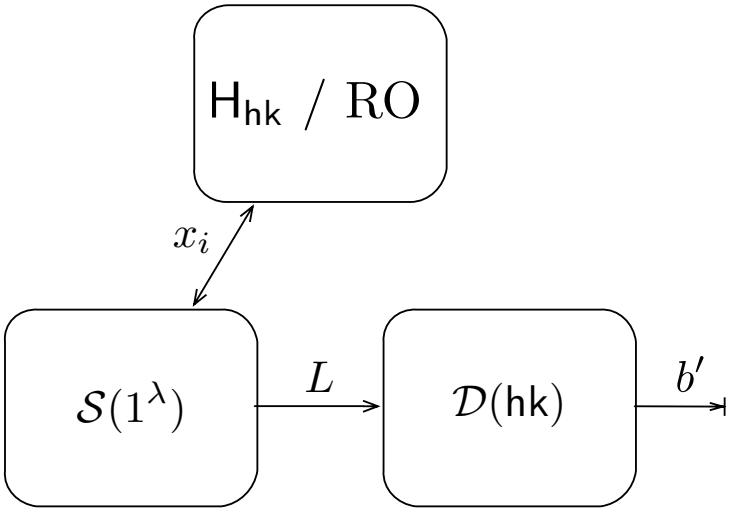
Second idea:

Restrict L: it must not leak any of $\mathcal{S}$'s queries.

Universal Computational Extractors (UCEs) [BHK13]



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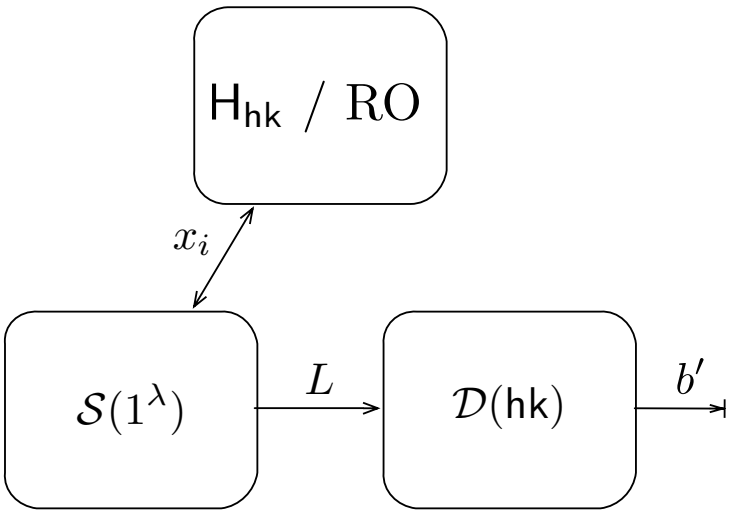

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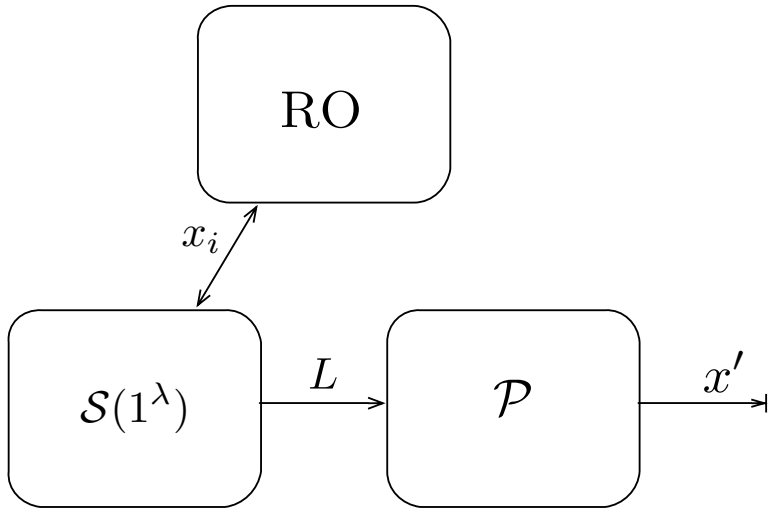

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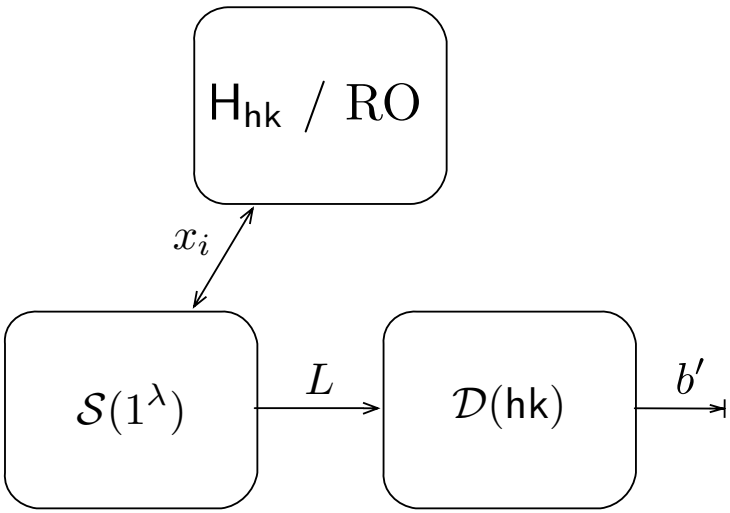

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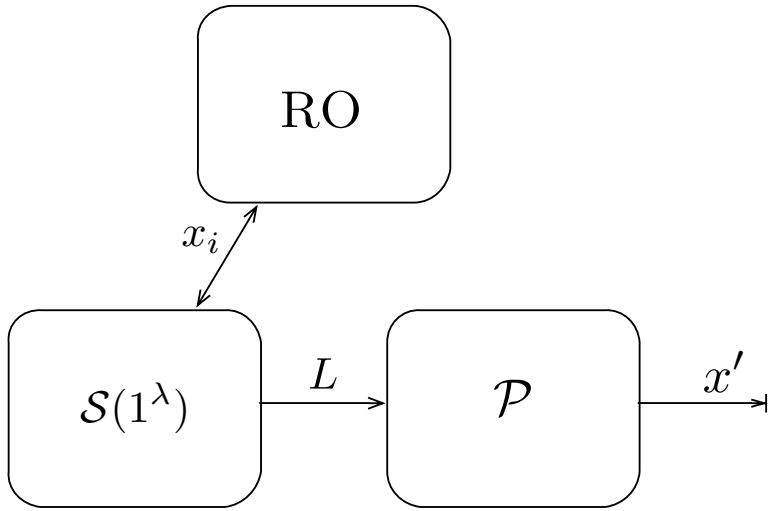

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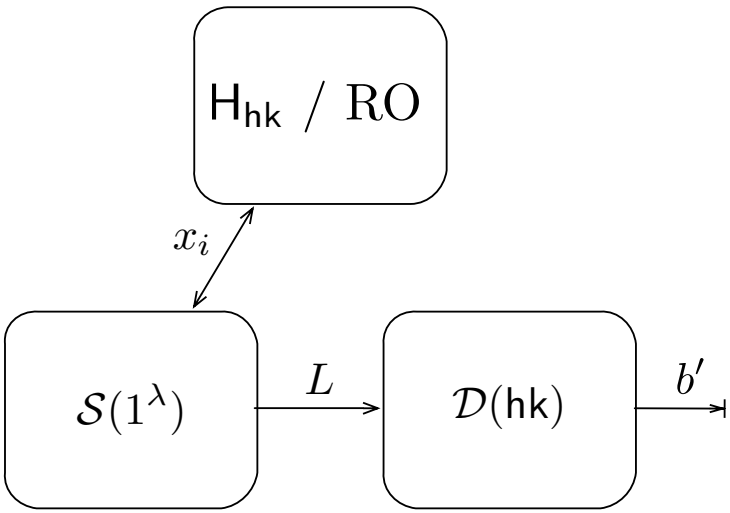

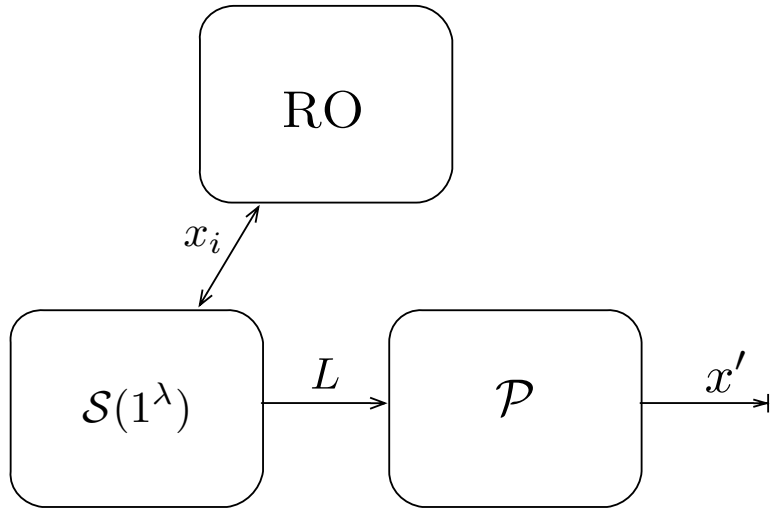

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Say H is **UCE secure** if $\mathbf{Adv}^{\mathrm{uce}}_{\mathsf{H},\mathcal{S},\mathcal{D}}(\lambda)$ is negl. for any **unpredictable** $\mathcal{S}$.

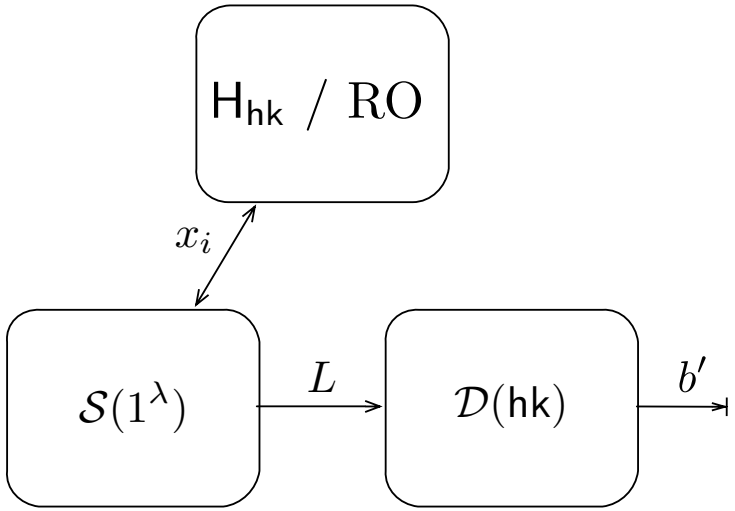
Applications of UCE [BHK13]

UCE-secure hash functions can instantiate the RO in:

- Deterministic public-key encryption (D-PKE)
- Message-locked encryption (MLE)
- Selective related-key and key-dependent message security
- Point-function obfuscation
- Proofs of storage
- Poly-many hard-core bits
- OAEP, garbling schemes, . . .

UCEs model many RO-like properties.

Shortcomings of UCEs



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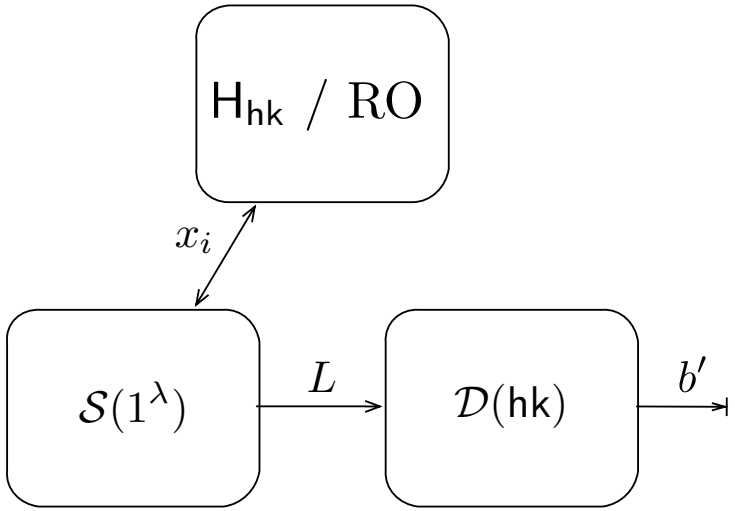

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Can we overcome these?

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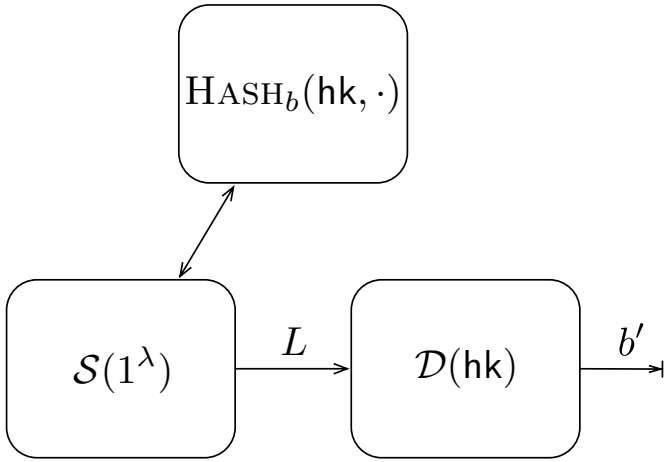
To what extent can we build on this view of UCEs to formulate

A more general framework modeling
a wider class of RO-like properties for hash functions.

Interactive Computational Extractors (ICEs)



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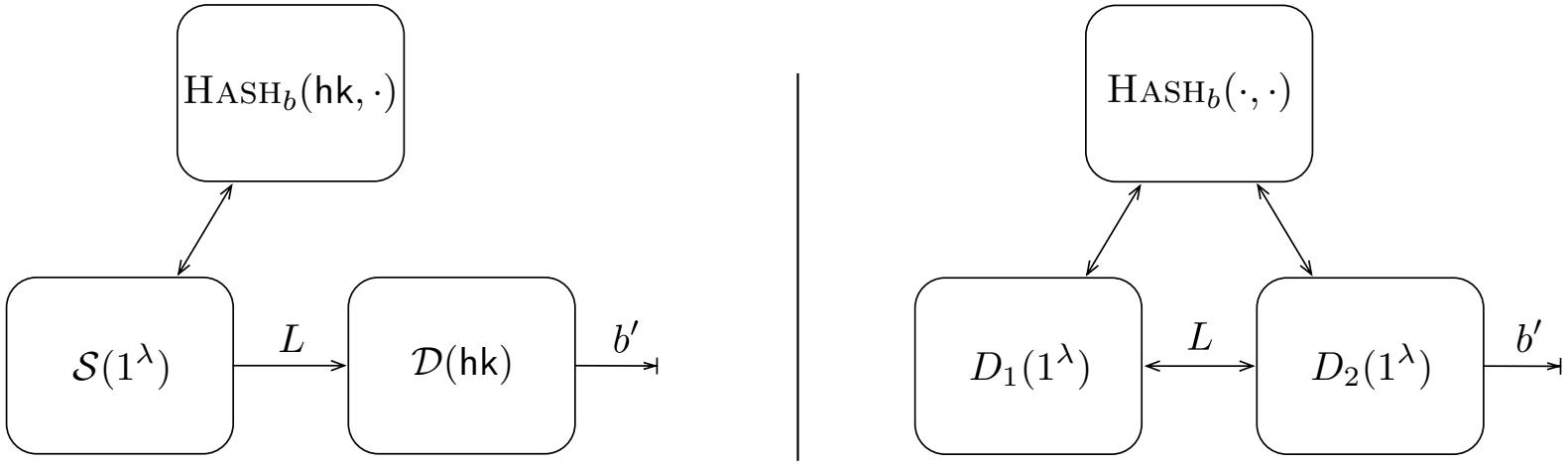

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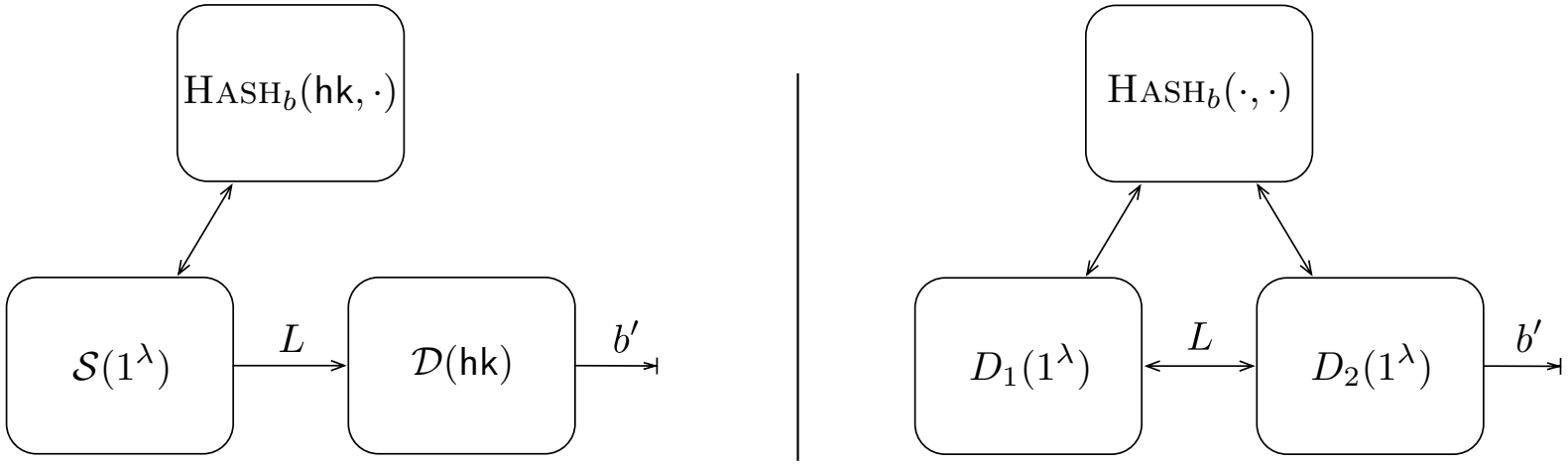

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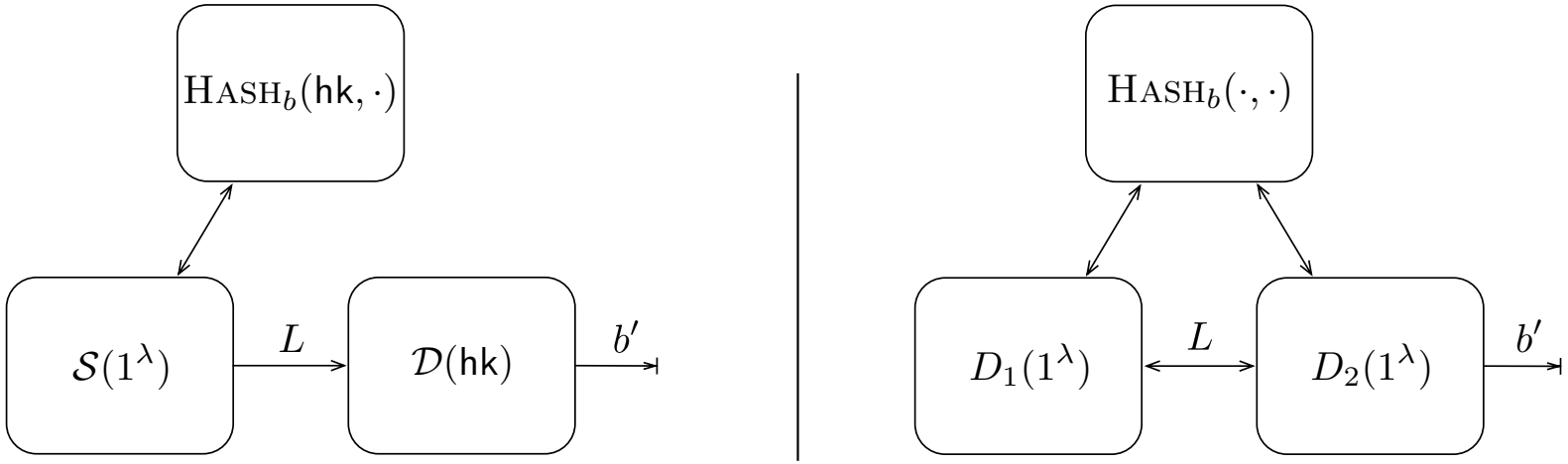

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$$x \leftarrow\!\!\$\ \mathcal{P}(\mathsf{View}(D_i)) \quad \text{st.} \quad x \in \mathsf{Qrys}(D_1) \cup \mathsf{Qrys}(D_2)\ ,$$

where

$$\begin{aligned}\mathsf{View}(D_i) &:= \text{Everything that } D_i \text{ sees} \\ &= \mathsf{Coins}(D_i) + \mathsf{InLeakage}(D_i) + \text{Hash values}\ .\end{aligned}$$

Example Application: RKA Security

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Full RKA: Via a new ICE notion (see upcoming full version).

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  - Non-adaptive RKA/KDM security
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- Semi-adaptive split KDM security of the BRS scheme
- Correlated-input hashing
- Foundational primitives:
  - (weak) PRFs,
  - Randomness extractors,
  - One-way security for polynomial regularity, . . .

Feasibility I: VIL-ROM

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$\mathsf{H}^{\mathcal{RO}}(\mathsf{hk}, M) := \mathcal{RO}(\mathsf{hk}|M)$ is ICE secure against computationally unpredictable (D_1, D_2).

Feasibility II: FIL-ROM

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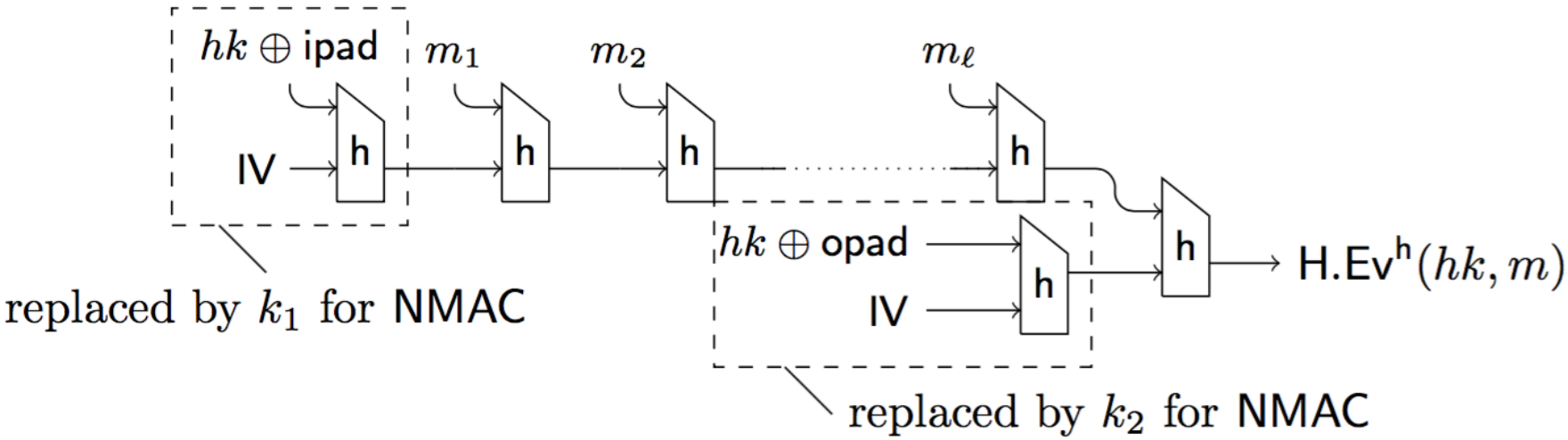

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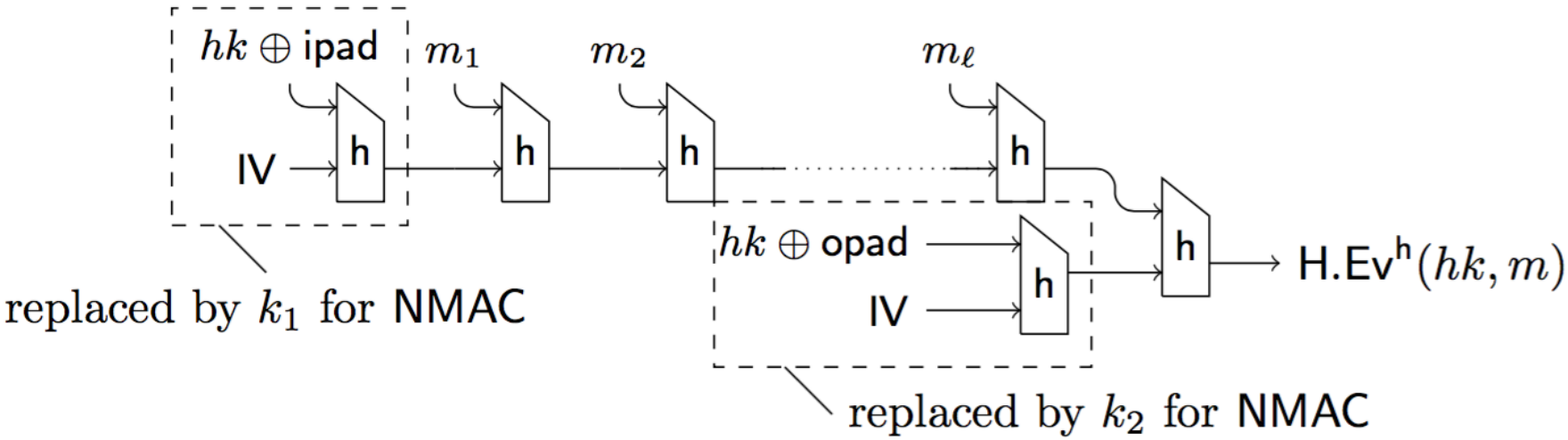

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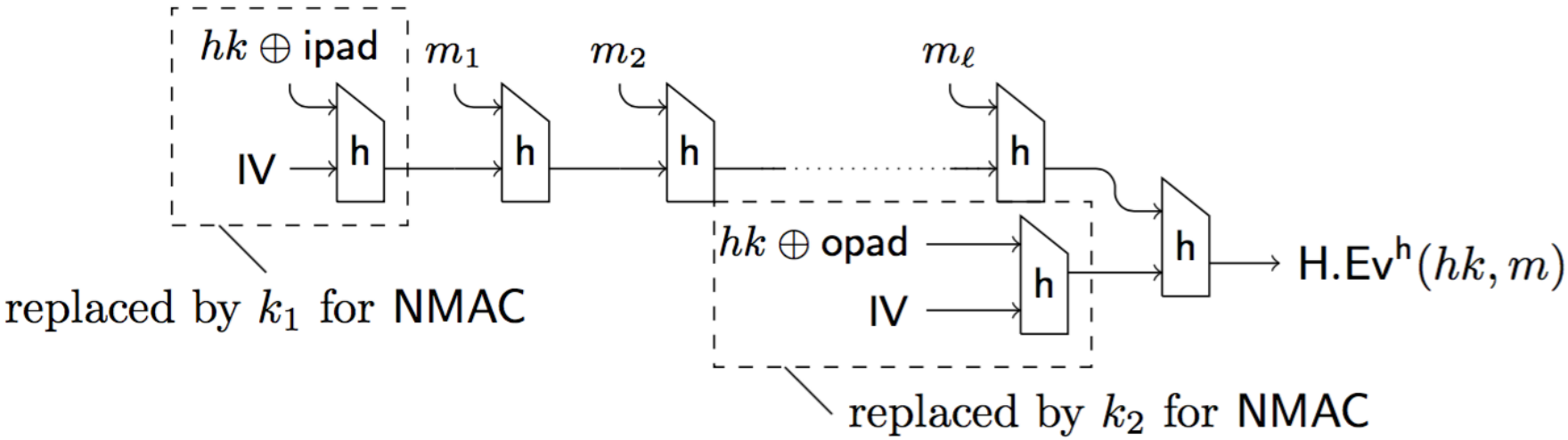

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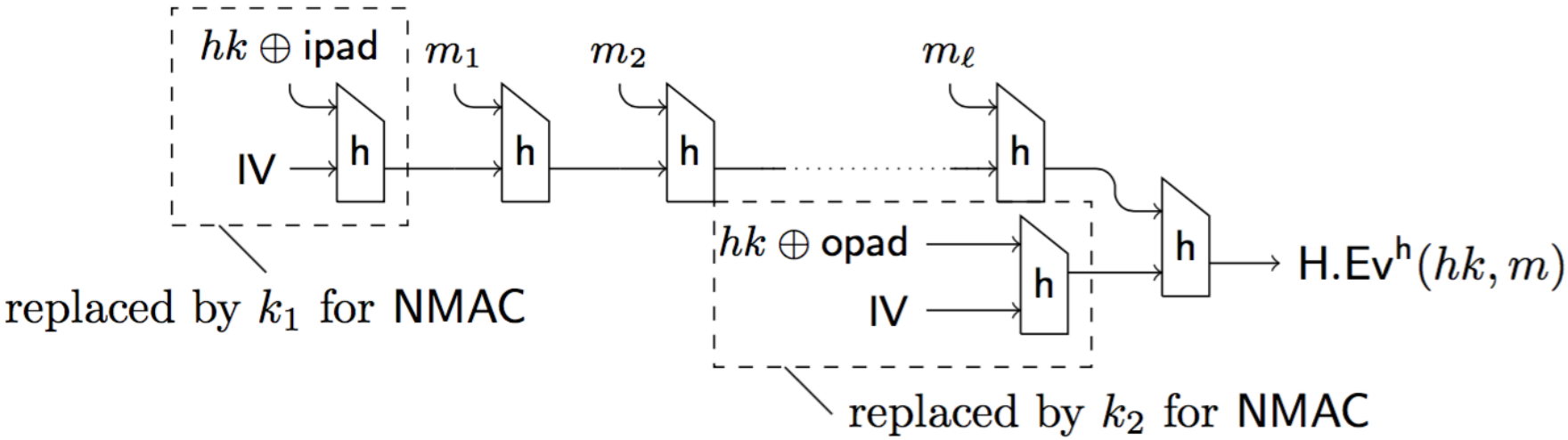

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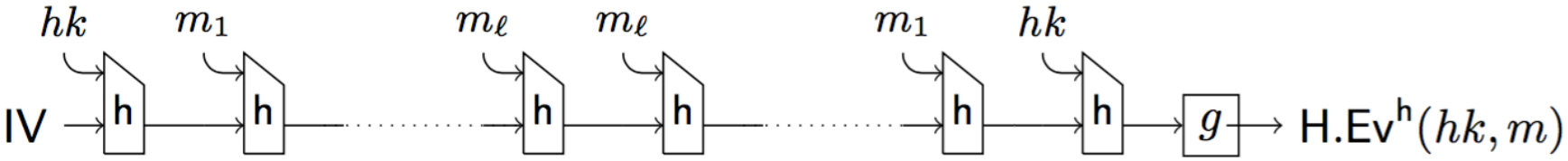

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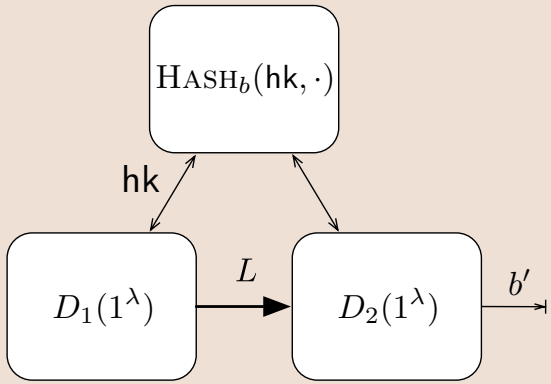

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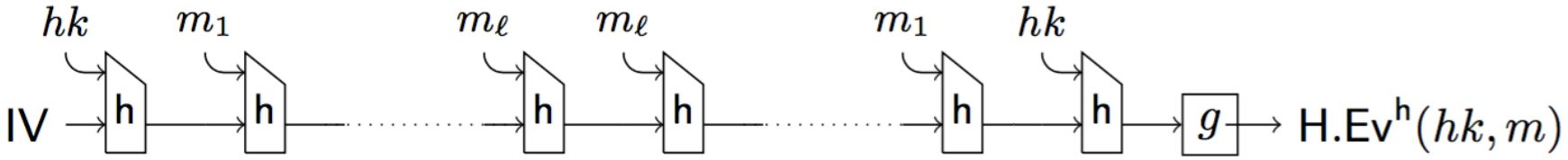

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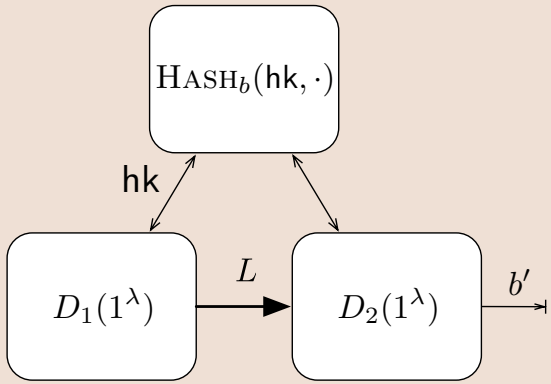

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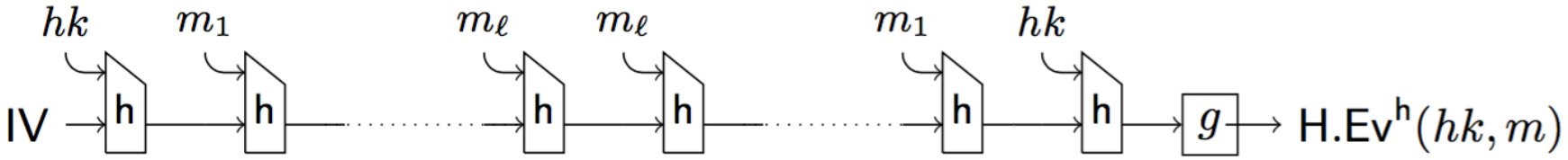

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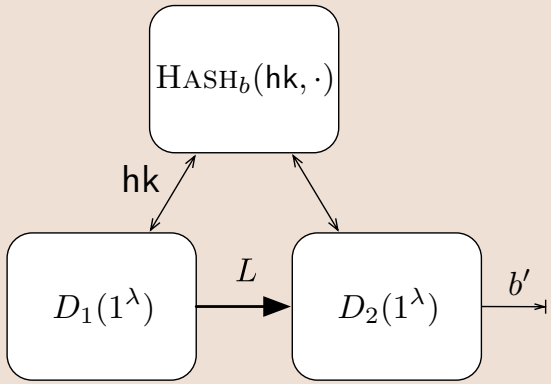

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- Shows multi-pass hash functions can provide extra security over their single-pass counterparts.

What about Full ICE in FIL-ROM?

Consider a message

$$M := \underbrace{[m_1, m_2]}_{M_1} \mid \underbrace{[m_3, m_4]}_{M_2} \mid \cdots \mid \underbrace{[m_{2n-1}, m_{2n}]}_{M_n}$$

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Construct all half-block pairs:

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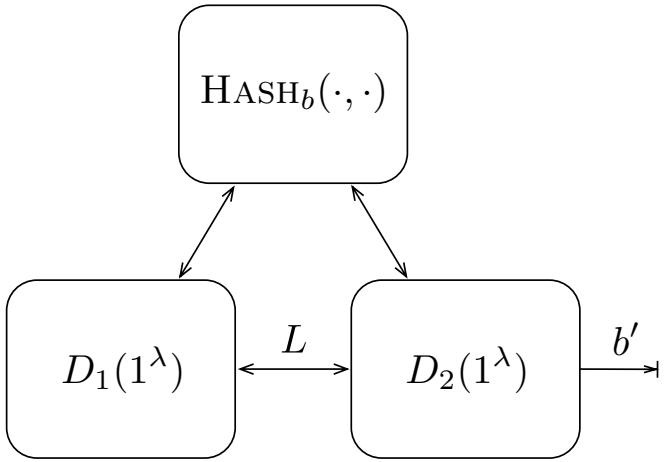
Conjecture

MixHash is fully ICE secure in the FIL-ROM.

Final Thoughts

What this talk was about:

A new security model capturing many RO-like properties.



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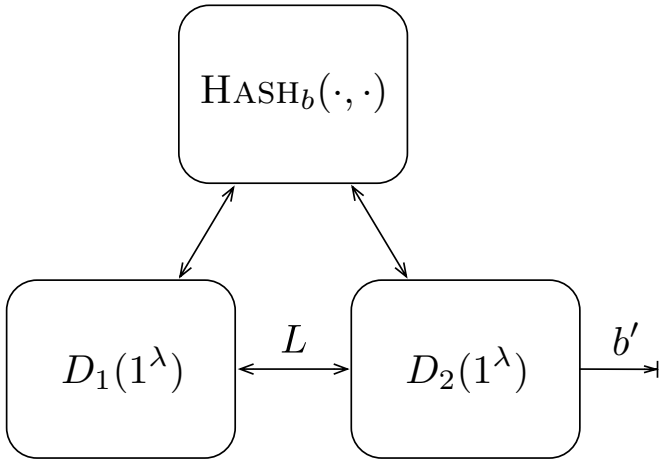

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- In particular, are there extensions that get us up to full KDM security?
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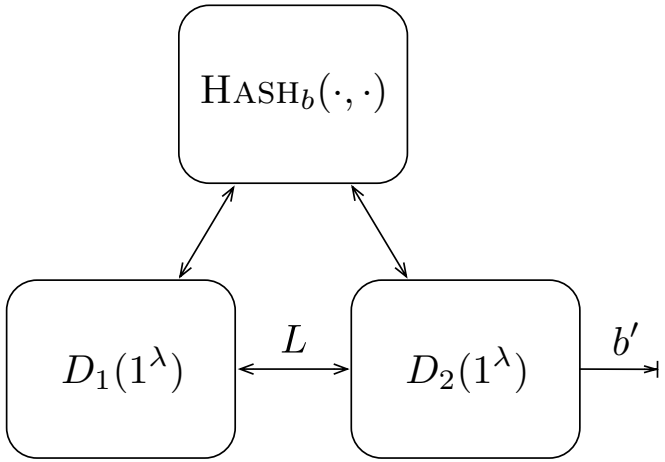

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Thank you.