Note on Impossible Differential Attacks

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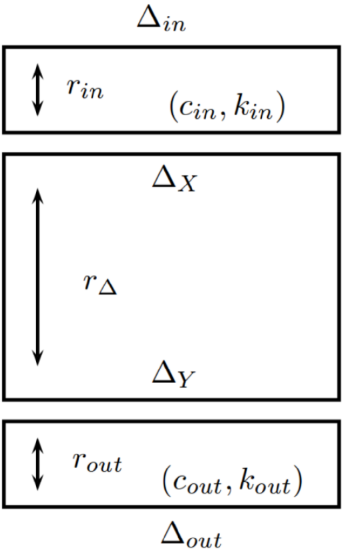
Outline

Outline for section 1

Impossible Differential Cryptanalysis



Setup

top $P[\Delta_{in} \to \Delta_X] = 2^{-c_{in}}$

middle $P[\Delta_X \to \Delta_Y] = 0$

bottom $P[\Delta_{out} \to \Delta_Y] = 2^{-c_{out}}$

Main idea

If a candidate key partially encrypts/decrypts a given pair to an impossible differential then this key is wrong.

Early Abort Technique Algorithm

Let $k_1, k_2, \ldots, k_b$ be key bits $k_{in} \cup k_{out}$ and σ a permutation.

Early abort

- Discard pairs which cannot follow the impossible differential
- Guess $k_{\sigma(1)}$
- Partially encrypt/decrypt pairs and discard pairs which cannot follow the impossible differential
- Guess $k_{\sigma(2)}$

$$\vdots$$

- Guess $k_{\sigma(b)}$
- Partially encrypt/decrypt pairs and discard pairs which cannot follow the impossible differential
- If all pairs have been discarded then perform an exhaustive search over remaining key bits.

Early Abort Technique Algorithm

Early abort without final exhaustive search - complexity

$$T_\sigma \geq \sum_{1 \leq i \leq b} 2^{|k_{\sigma(1)} \cup \ldots \cup k_{\sigma(i)}| - \sum_{1 \leq j < i} r_j^\sigma} \cdot N \cdot C'_E$$

- N: number of pairs
- r_i^σ: proportion of pairs discarded at step i
- C'_E: ratio of the cost of partial encryption to full encryption

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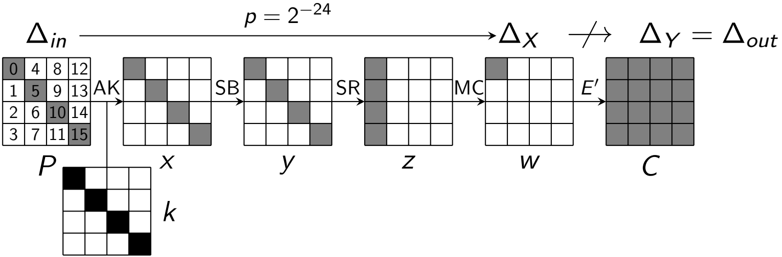
Boura *et al*'s assumption (ASIACRYPT 2014):

$$\min_\sigma T_\sigma \approx (1 + 2^{|k_{in} \cup k_{out}| - c_{in} - c_{out}}) \cdot N \cdot C_E'$$

Outline for section 2

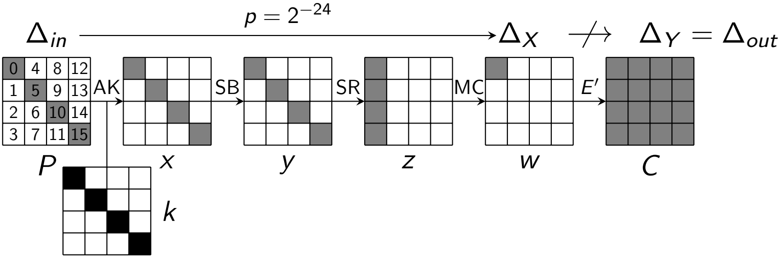
Toy Cipher

- First round based on AES



How key schedule relations do affect time complexity?

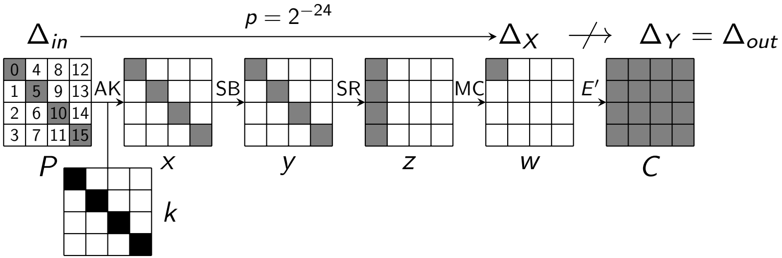
Independent Subkeys



- k_0, k_5, k_{10} and k_{15} independent
- Boura *et al*'s formula:

$$(1 + 2^{|k_{in}| - c_{in}}) \cdot N \cdot C'_E = (1 + 2^{32-24}) \cdot N \cdot 4S_E^{-1} \approx 2^{10} \cdot N \cdot S_E^{-1}$$

Independent Subkeys



Early abort technique:

- Guess k_0 and keep only pairs for which transitions $\Delta x_5 \to \Delta y_5$, $\Delta x_{10} \to \Delta y_{10}$ and $\Delta x_{15} \to \Delta y_{15}$ are possible
- Guess k_5 and keep only pairs satisfying $\Delta x_5 \to \Delta y_5$
- Guess k_{10} and keep only pairs satisfying $\Delta x_{10} \to \Delta y_{10}$
- Guess k_{15} and keep only pairs satisfying $\Delta x_{15} \to \Delta y_{15}$

Real complexity:

$$(2^8 + 2^{8+8-3} + 2^{8+8+8-3-7} + 2^{8+8+8+8-3-7-7}) \cdot N \cdot S_E^{-1} \approx 2^{15.8} \cdot N \cdot S_E^{-1}$$

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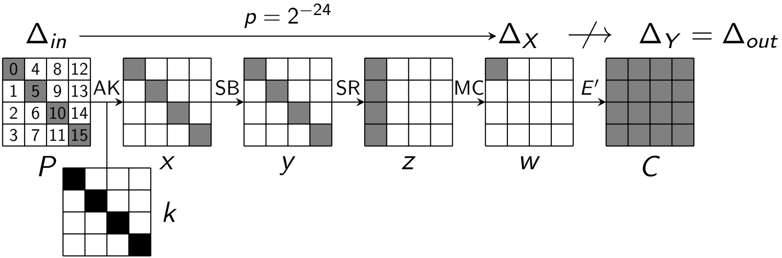

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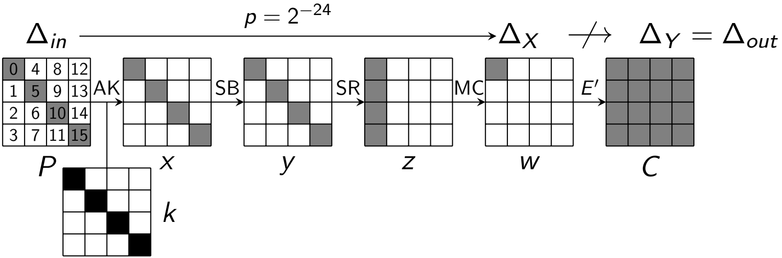
Related Subkeys I



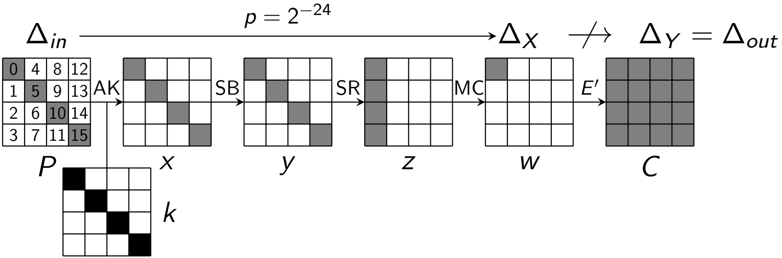
- one key schedule equation: $k_0 = k_5$
- Boura *et al*'s formula:

$$(1 + 2^{|k_{in}| - c_{in}}) \cdot N \cdot C'_E = (1 + 2^{24-24}) \cdot N \cdot 4S_E^{-1} = 2^3 \cdot N \cdot S_E^{-1}$$

- Real complexity:

$$(2^8 + 2^{8-3} + 2^{8+8-3-7} + 2^{8+8+8-3-7-7}) \cdot N \cdot S_E^{-1} \approx 2^{8.9} \cdot N \cdot S_E^{-1}$$

Related Subkeys II



- one key schedule equation: $k_0 \oplus k_5 \oplus k_{10} \oplus k_{15} = 0$
- Boura *et al*'s formula:

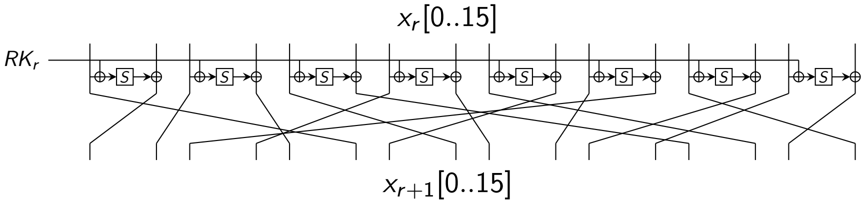
$$(1 + 2^{|k_{in}| - c_{in}}) \cdot NC'_E = (1 + 2^{24-24}) \cdot N \cdot 4S_E^{-1} = 2^3 \cdot N \cdot S_E^{-1}$$

- Real complexity:

$$(2^8 + 2^{8+8-3} + 2^{8+8+8-3-7} + 2^{8+8+8-3-7-7}) \cdot N \cdot S_E^{-1} \approx 2^{14.6} \cdot N \cdot S_E^{-1}$$

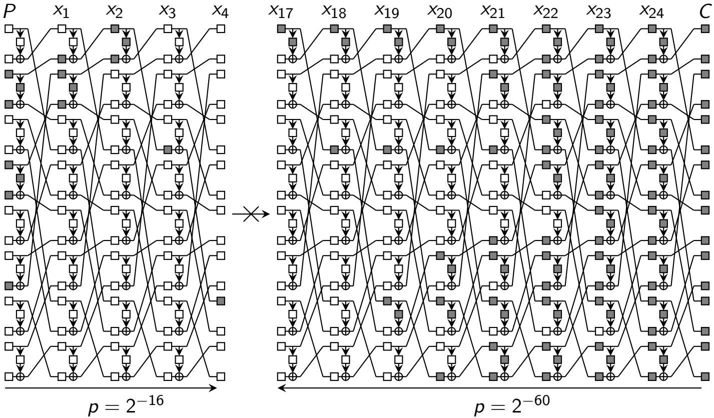
Outline for section 3

TWINE



- Nibble-oriented Feistel
- state size: 64 bits (16 4-bit branches)
- 2 key sizes: 64 and 128 bits

Biryukov et al's attack (FSE 2015)



- 52 subkey nibbles involved but only 2^{124} possible values

Methodology

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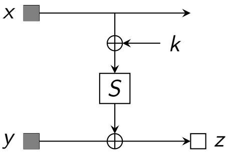
If between two guesses no pairs are discarded then the order in which they are guessed does not matter.

When do pairs are discarded?

Discarding pairs



Proportion of pairs:

- Δx, Δy: probability of transition $\Delta x \to \Delta y \approx 2^{-1}$
- Δx, Δy, $x \oplus k$: 2^{-4}

Exhausting Early Abort Technique

- Biryukov *et al*'s attack:
  - 19 tuples (x, y, z)
    $\rightarrow$ 19 tuples $(\Delta x, \Delta y)$ + 19 tuples $(\Delta x, \Delta y, x \oplus k)$
  - Easy to determine corresponding subkey nibbles
  - But brute force still infeasible:

$$(19 + 19)! = 38! \approx 2^{148}$$

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- Search:
  - $K_i \subseteq K_j \Rightarrow$ guess K_i before K_j
  - Generic formula:

$$\text{Best}(K_1, \ldots) = \min_i \left(\text{Best}(K_1, \ldots, K_{i-1}, K_{i+1}, \ldots) + 2^{|K_1 \cup \ldots| - \sum_{j \neq i} r(K_j)}\right)$$

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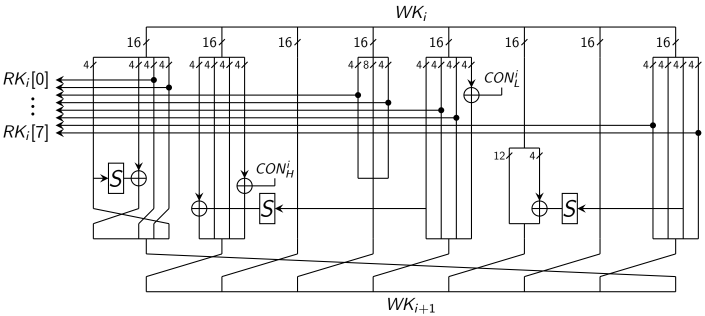
How to compute $2^{|K_1 \cup \ldots|}$?

TWINE-128 Key Schedule



- Shape of key schedule equations:

$$\bigoplus \alpha_i k_i \oplus \beta_i S(k_i) = \gamma,$$

where α_i's, β_i's and γ are constant

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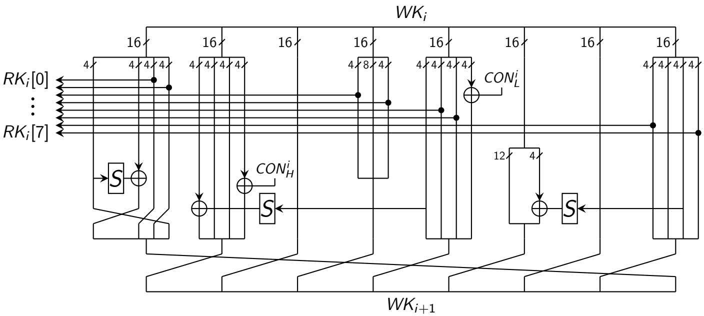

- Shape of key schedule equations:

$$\bigoplus \alpha_i k_i \oplus \beta_i S(k_i) = \gamma,$$

where α_i's, β_i's and γ are constant
- Use Derbez *et al*'s tool (FSE 2013)

Result

- Computation of real complexity: 1h on personal computer
- Result:

$$\min_{\sigma} T_{\sigma} \geq 2^{54} \cdot N_{\alpha} \cdot C_{E'}$$

- Time complexity of whole attack higher than:

$$C_{N_{\alpha}} + \alpha \cdot 2^{127.6} + 2^{128-\alpha}$$

- Higher than 2^{128} for all $\alpha > 0$.

Outline for section 4

Conclusion

In this paper:

- Boura *et al*'s formula too optimistic
  - reaching it is, if not impossible, very tricky
- Construction of simple counter-examples:
  $\longrightarrow$ deviation up to a factor $2^{11.6}$
- Algorithm computing real complexity for TWINE
  - complexity of Biryukov *et al*'s attack higher than 2^{128}
  - applicable to more ciphers

Open problems:

- Improve the formula
- Find an example with time complexity smaller than expected

Thanks

Thank you for your attention!