

# New Bounds for Keyed Sponges with Extendable Output: Independence between Capacity and Message Length

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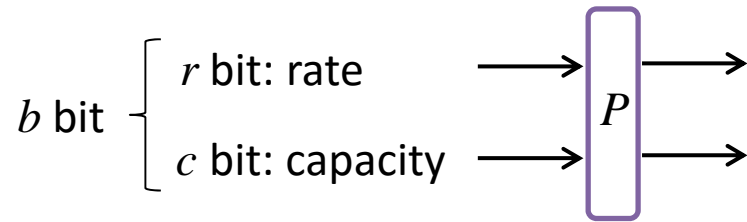
Mitsubishi Electric Corporation

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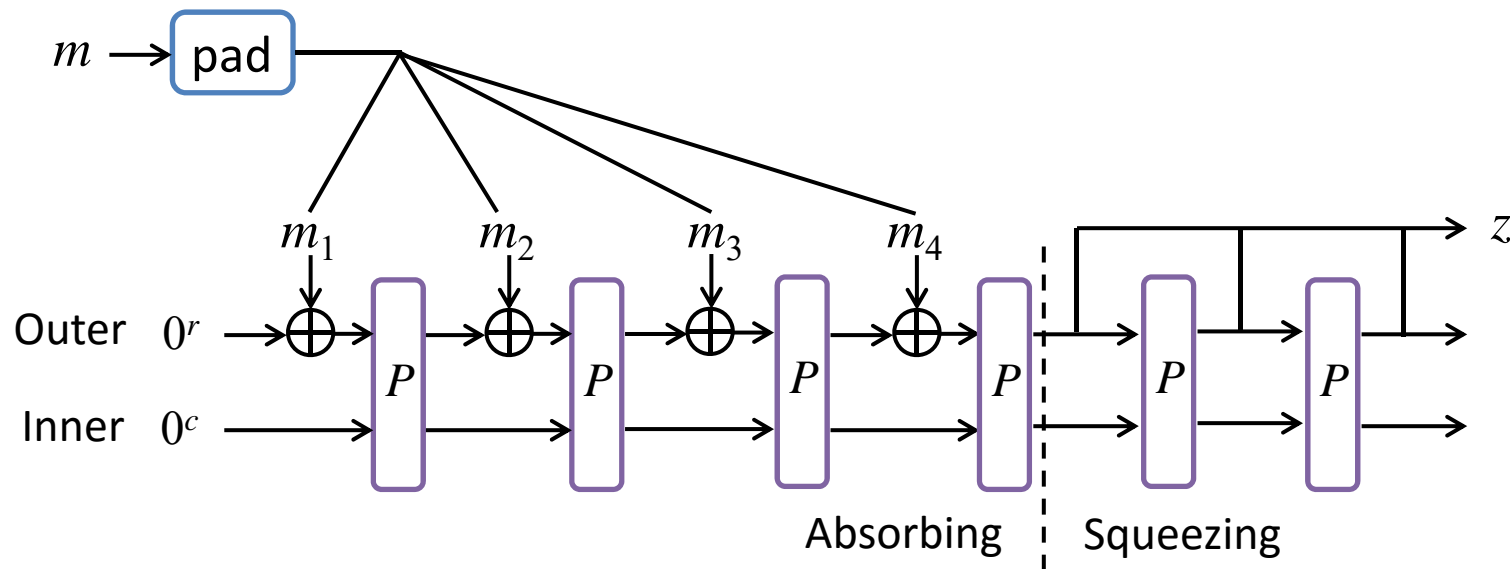
NTT Secure Platform Laboratories

# Sponge

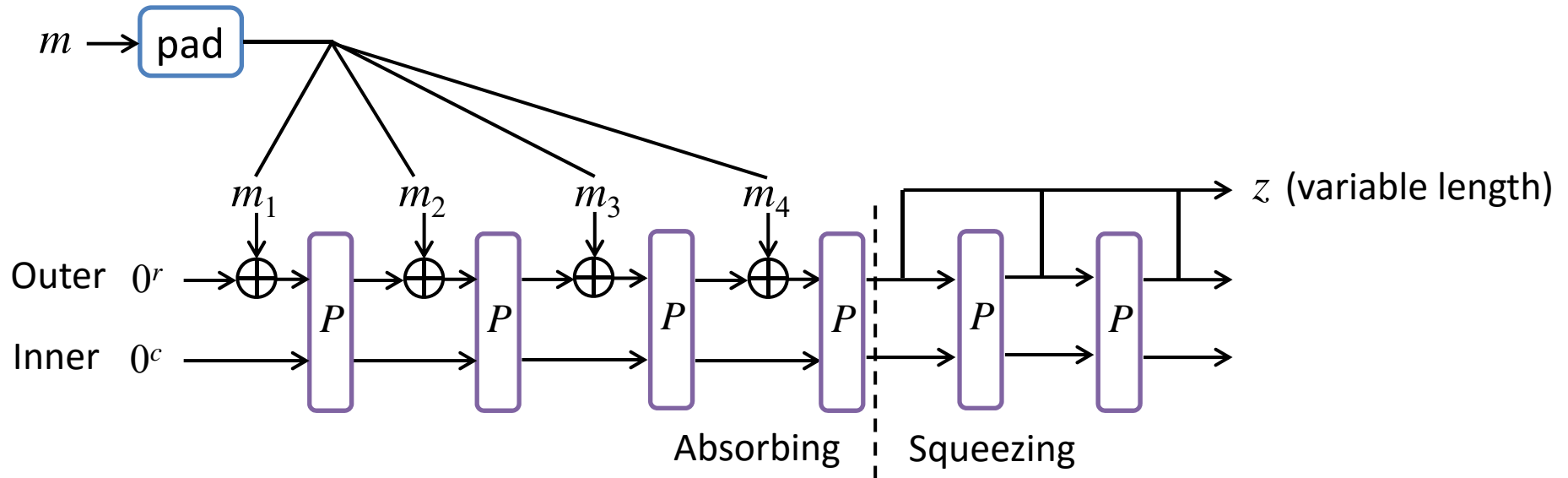
- Based on a permutation  $P: \{0,1\}^b \rightarrow \{0,1\}^b$



- Iterate permutation  $P$



# Sponge with Extendable Output

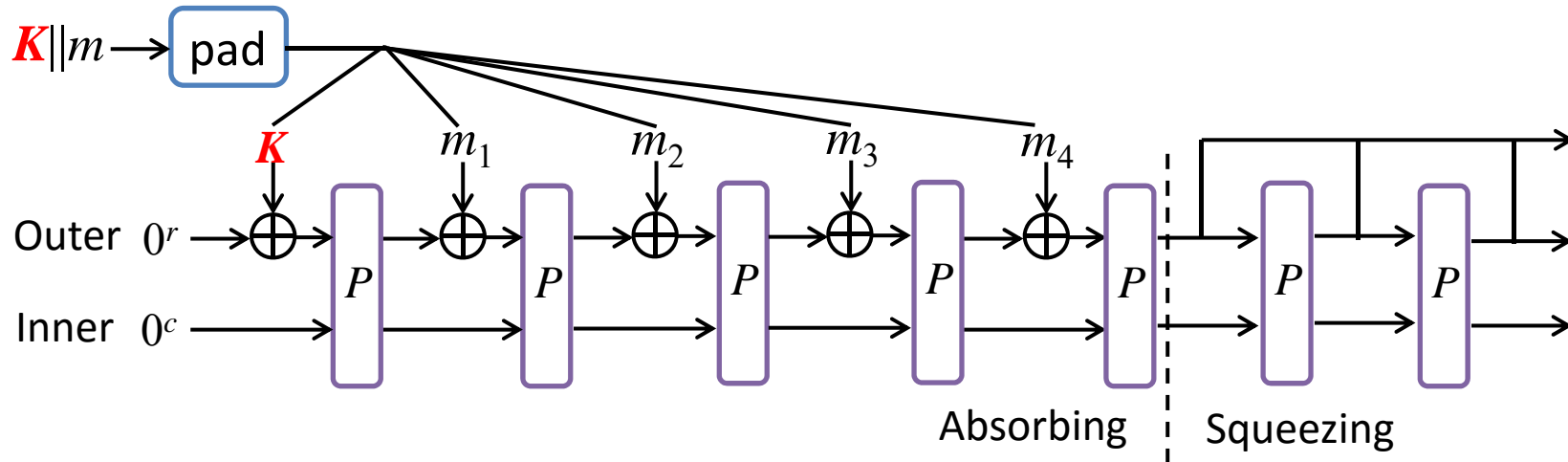


## ■ Sponge with extendable output

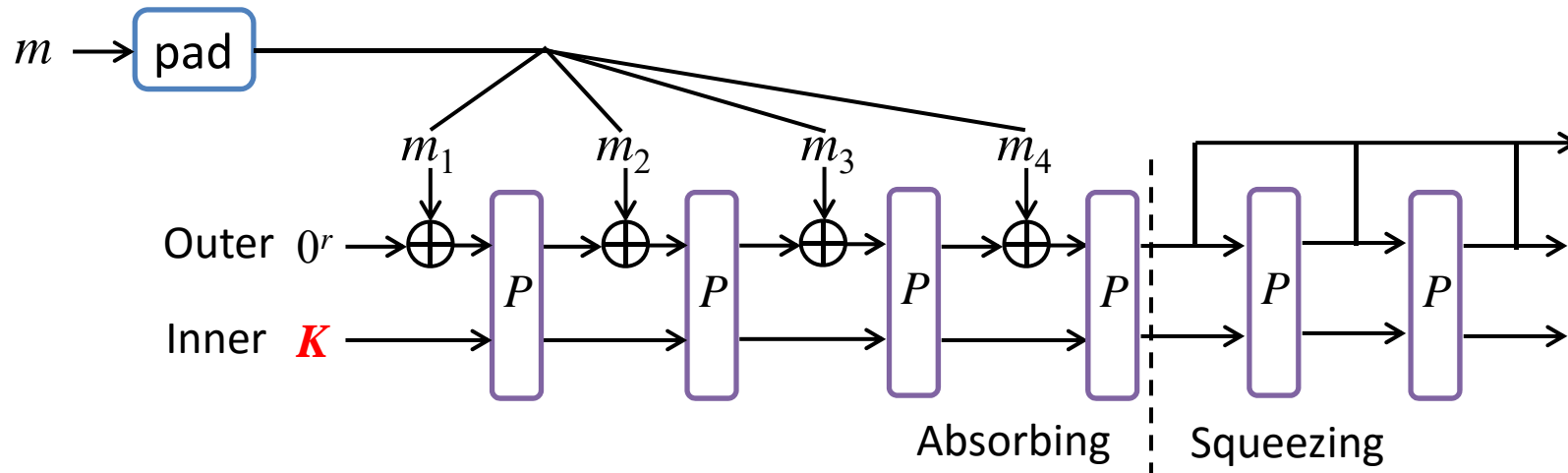
- used as the constructions of SHAKE128, SHAKE256 in FIPS 202
- Designed to construct key derivation functions

# Keyed Sponges with Extendable Output

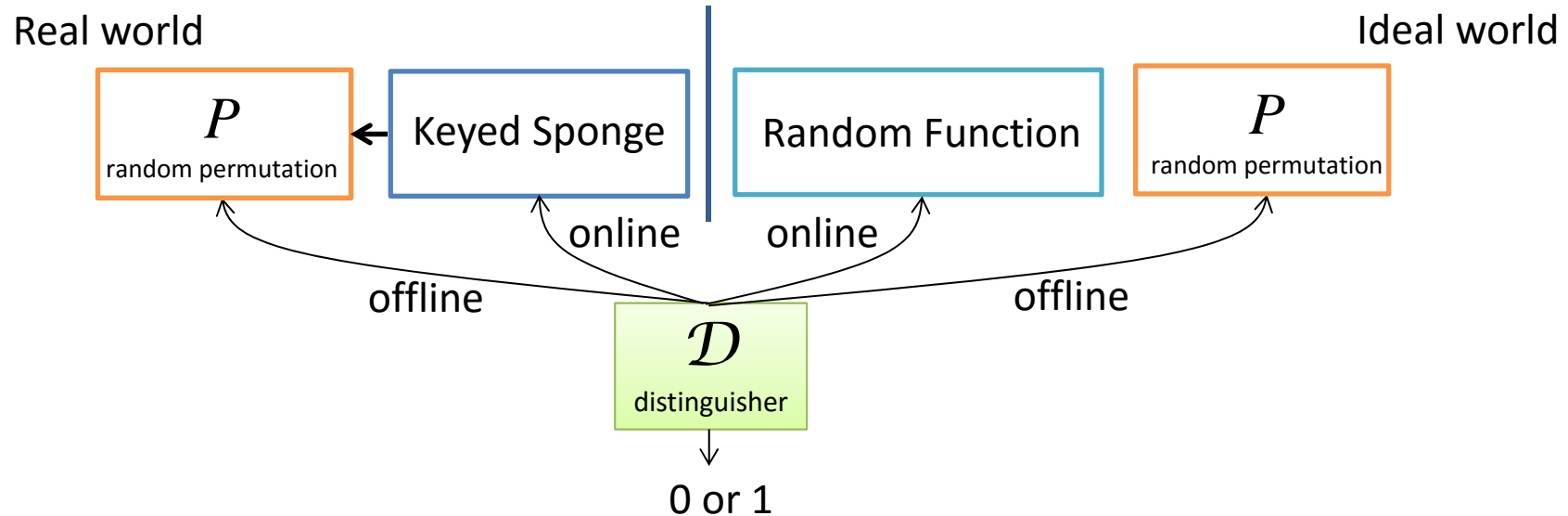
## Outer Keyed Sponge (OKS)



## Inner Keyed Sponge (IKS)



# PRF-Security of Keyed Sponges



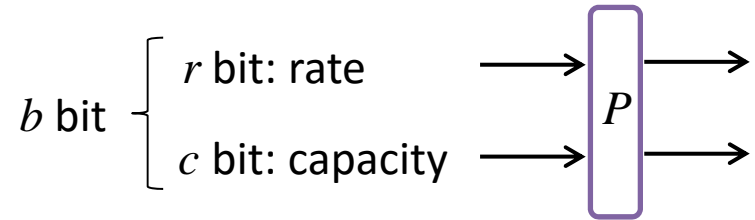
## ■ Parameters

- $\ell$  is the maximum input length to a keyed sponge in blocks
- $q$  is the number of online queries
- $Q$  is the number of offline queries

# Previous PRF-Security Bounds

■ There are two terms:

- capacity term e.g.,  $(\ell q + Q)^2 / 2^c$ ,  $(\ell q)^2 / 2^c$ , etc
- $b$ -term e.g.,  $(\ell q)^2 / 2^b$ ,  $\ell q Q / 2^b$ , etc



■ Since  $c < b$  (e.g.,  $b=1600$ ,  $c=256$ ,  $512$ ),

the capacity term becomes a dominant term in the PRF-security bounds

■ The capacity term has been improved

	Capacity term	Target
Bertoni et al. (Indiff.) (EUROCRYPT'08)	$(\ell q + Q)^2 / 2^c$	OKS
Andreeva et al. (FSE'15)	$((\ell q)^2 + \mu Q) / 2^c$ $\mu$ is multiplicity: $2\ell q / 2^r \leq \mu \leq 2\ell q$	OKS and IKS
Gazi et al. (CRYPTO'15)	$(\ell q + q^2 + qQ) / 2^c$	$\exists$ attack with prob. $(q^2 + qQ) / 2^c$ If $\ell \leq q$ or $\ell \leq Q$ then the bound is tight
Mennink et al. (ASIACRYPT'15)	$(\ell q^2 + \mu Q) / 2^c$ $\mu$ is multiplicity: $2\ell q / 2^r \leq \mu \leq 2\ell q$	IKS [*]

[\*] Support the constructions with full message state absorption 6

# Comparison and Our Result

**Open problem for keyed sponges with extendable output:**  
Improve the capacity terms so that they become tight terms

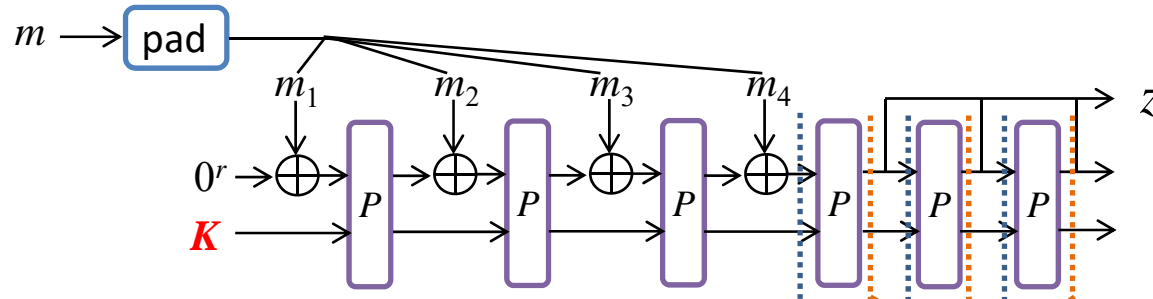
	IKS	OKS	Extendable output	Tightness of c-term
Bertoni et al.		✓	✓	
Andreeva et al.	✓	✓	✓	
Gazi et al.		✓		✓
Mennink et al.	✓		✓	
<b>This paper</b>	✓	✓	✓	✓

Today's talk

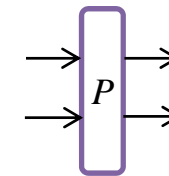
# PRF-Security Proof of IKS

## Real world

online query

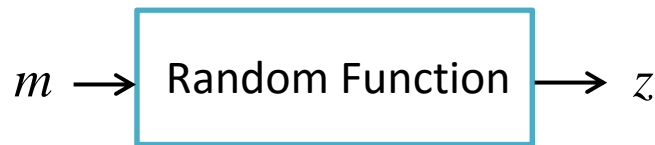


offline query



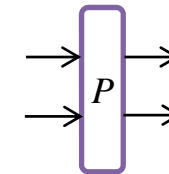
## Ideal world

online query



(almost) b-bit random values

offline query



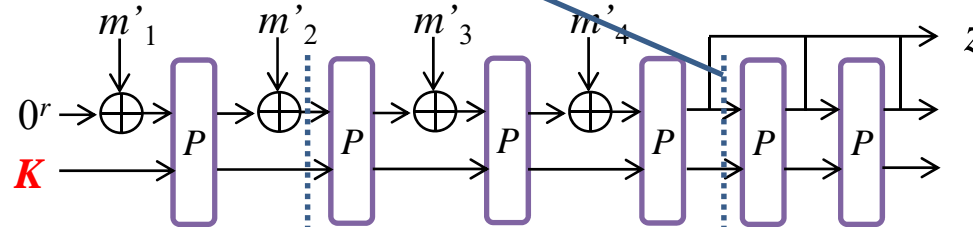
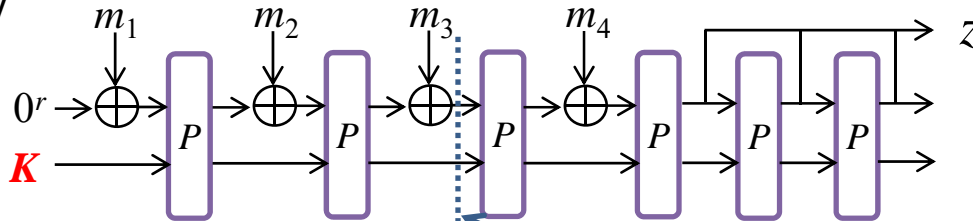
- For any query to a random function, the response is randomly drawn
- If all outputs of IKS can be seen as random values then the real world and the ideal world are indistinguishable
- If all inputs to  $P$  that produces outputs of IKS are new inputs then all outputs of IKS become (almost) random values
- The distinguishing prob.
  - $\leq$  The prob. that some input to  $P$  that produces the output of IKS is not new



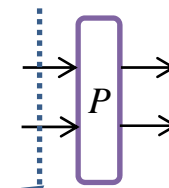
# PRF-Security Proof of IKS

## Real world

online query



offline query

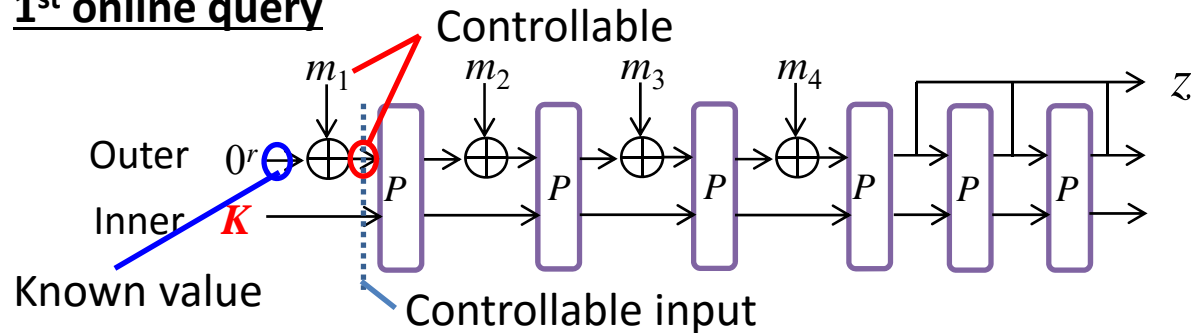


- The distinguishing prob.
  - $\leq$  The prob. that some input to  $P$  that produces the output is not a new input
  - $\leq$  The prob. that a collision in inputs to  $P$  occurs
- We categorize inputs to  $P$  in IKS into two sorts of inputs
  - Controllable input
  - Uncontrollable input

# Controllable Input and Uncontrollable Input

- Controllable input
  - The outer part can be controlled
  - The inner part cannot be controlled
- Uncontrollable input
  - The outer and inner parts cannot be controlled

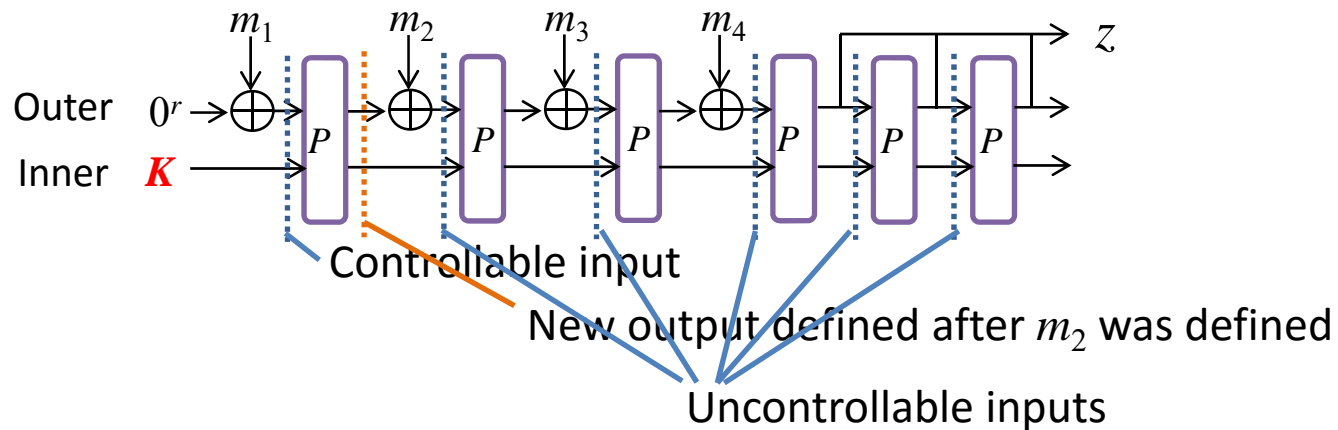
## 1<sup>st</sup> online query



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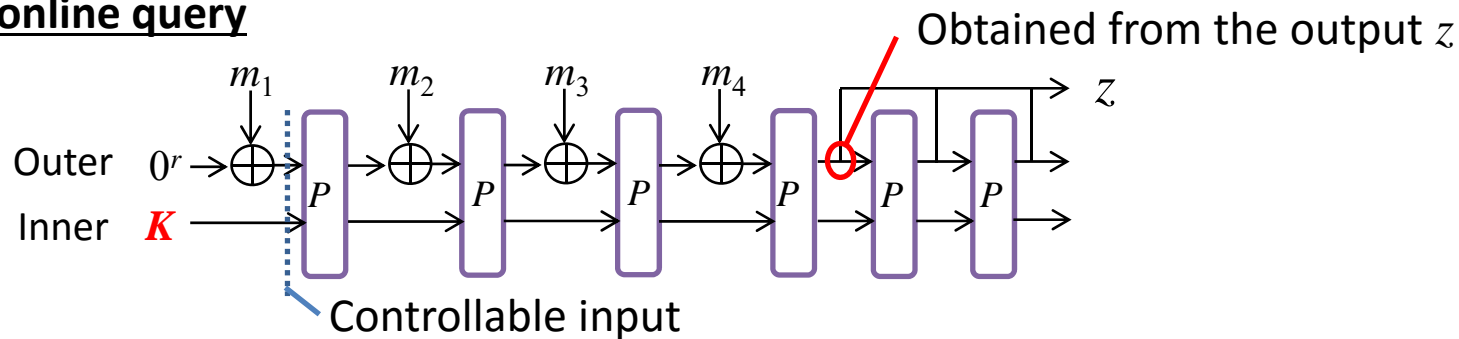
## 1<sup>st</sup> online query



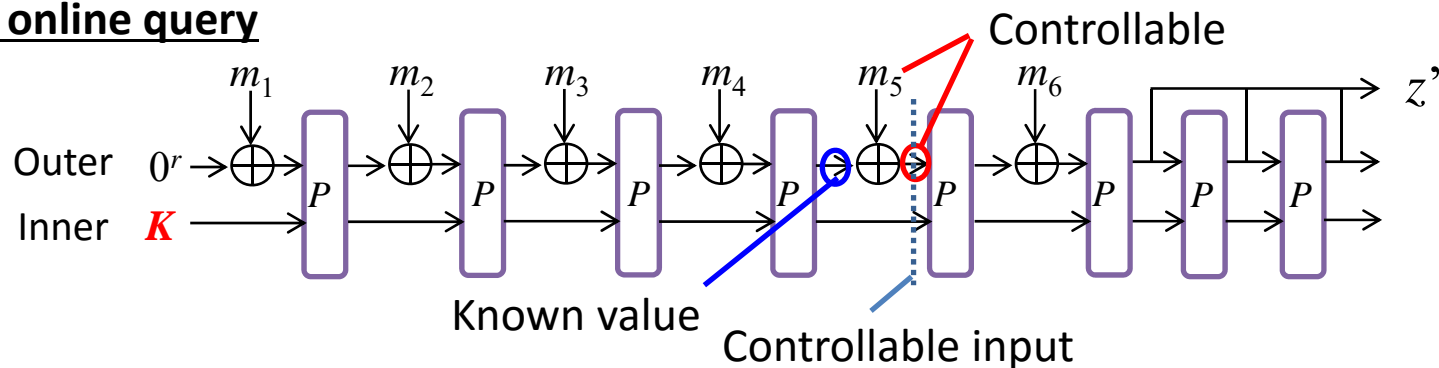
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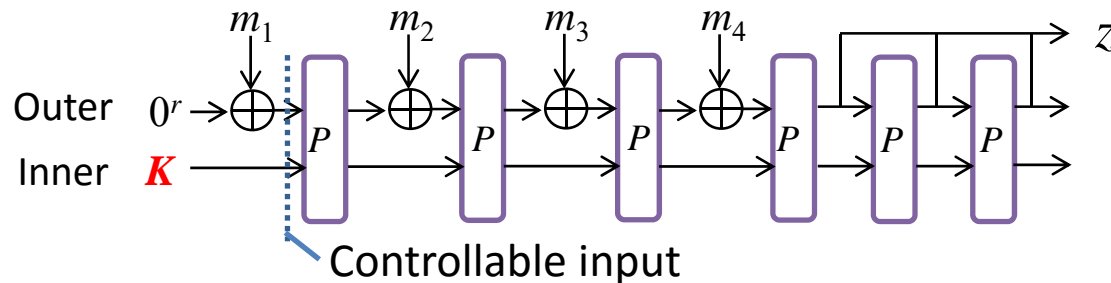
## 2<sup>nd</sup> online query



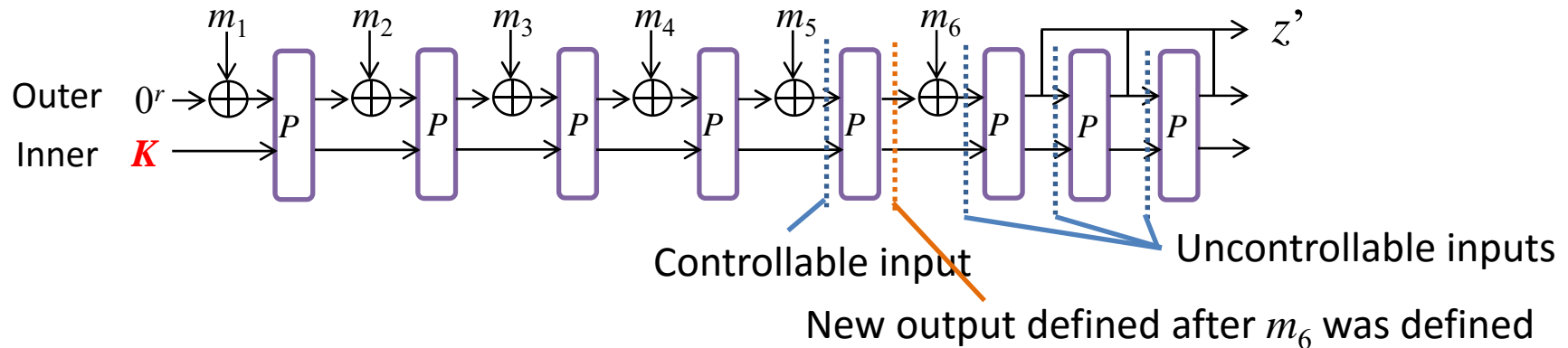
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## 1<sup>st</sup> online query



## 2<sup>st</sup> online query





# Controllable Input and Uncontrollable Input

- The features of controllable inputs and uncontrollable inputs

Inputs to $P$	Outer part (r bit)	Inner part (c bi)	Number
Controllable input	Not random	Random	$q$
Uncontrollable input	Random	Random	$\ell q$

# Bound of Distinguishing Probability

- The distinguishing prob.  $\leq$  The prob. that a collision in inputs to  $P$  occurs

Inputs to $P$	Outer part (r bits)	Inner part (c bits)	Number
Controllable input	Not random	Random	$q$
Uncontrollable input	Random	Random	$\ell q$
Input by offline query	Not random	Not random	$Q$

- The prob. that a collision in inputs to  $P$  occurs is bounded by the sum of probabilities of
  - collision in controllable inputs  $\Rightarrow q^2/2^c$
  - collision in uncontrollable inputs  $\Rightarrow (\ell q)^2/2^b$
  - collision between controllable inputs and uncontrollable inputs  $\Rightarrow \ell q^2/2^b$
  - collision between controllable inputs and inputs by offline queries  $\Rightarrow qQ/2^c$
  - collision between uncontrollable inputs and inputs by offline queries  $\Rightarrow \ell qQ/2^b$
- The distinguishing prob.  $\leq (q^2 + qQ)/2^c + (\ell q^2 + (\ell q)^2 + \ell qQ)/2^b$

Attack with prob.  $(q^2 + qQ)/2^c$



# Conclusion

	Capacity term	Target
Indifferentiability (EUROCRYPT'08)	$(\ell q + Q)^2 / 2^c$	OKS
Andreeva et al. (FSE'15)	$((\ell q)^2 + \mu Q) / 2^c$ $\mu$ is multiplicity: $2\ell q / 2^r \leq \mu \leq 2\ell q$	OKS and IKS
Gazi et al. (CRYPTO'15)	$(\ell q + q^2 + qQ) / 2^c$	OKS with single block output
Mennink et al. (ASIACRYPT'15)	$(\ell q^2 + \mu Q) / 2^c$ $\mu$ is multiplicity: $2\ell q / 2^r \leq \mu \leq 2\ell q$	IKS
<b>This paper</b>	<b><math>(q^2 + qQ) / 2^c</math> (Tight)</b>	<b>IKS and OKS</b>

Thank You!